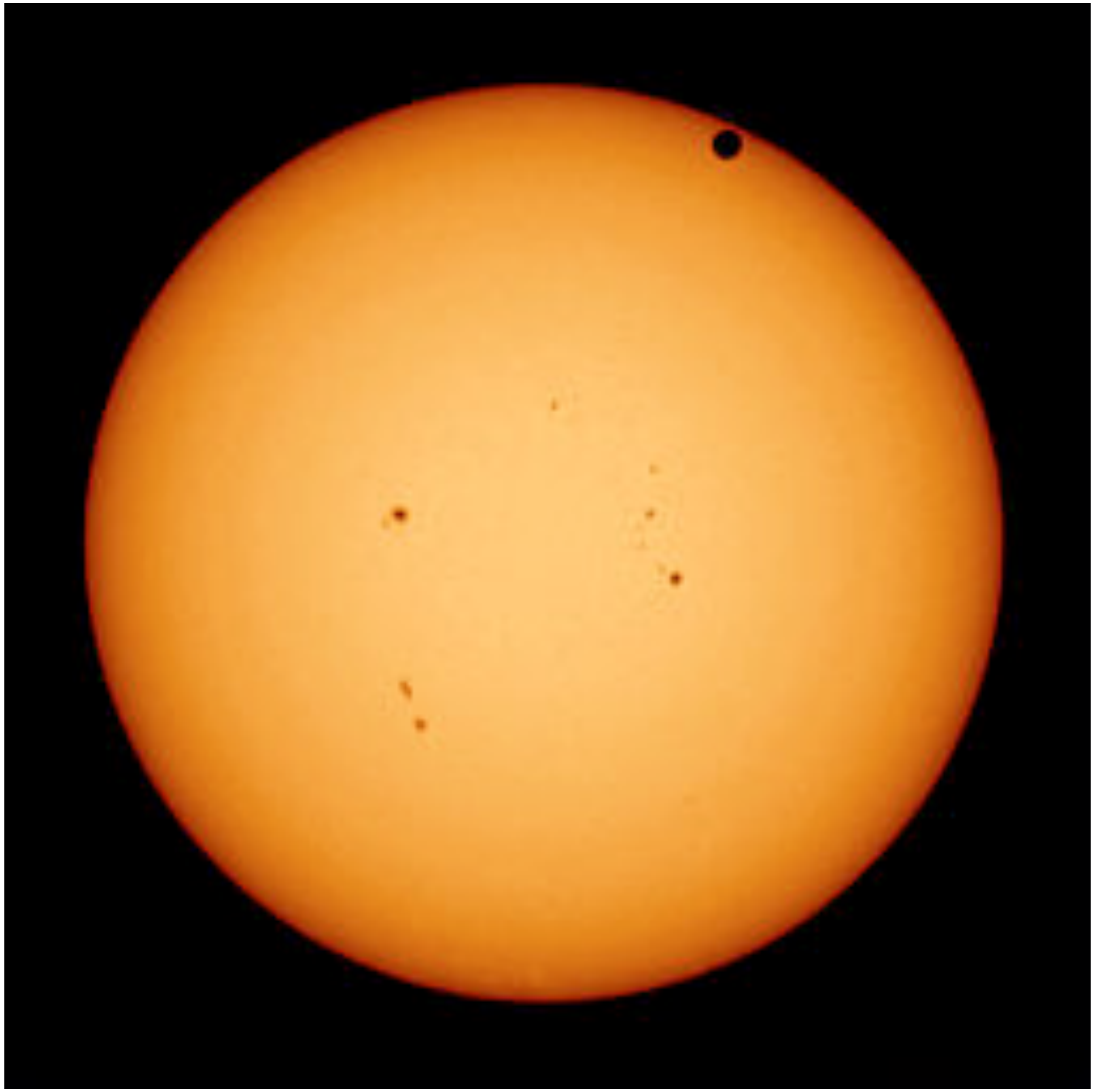
Lecture 4: Transit parameter fitting

Transit models

Markov-chain Monte-Carlo methods

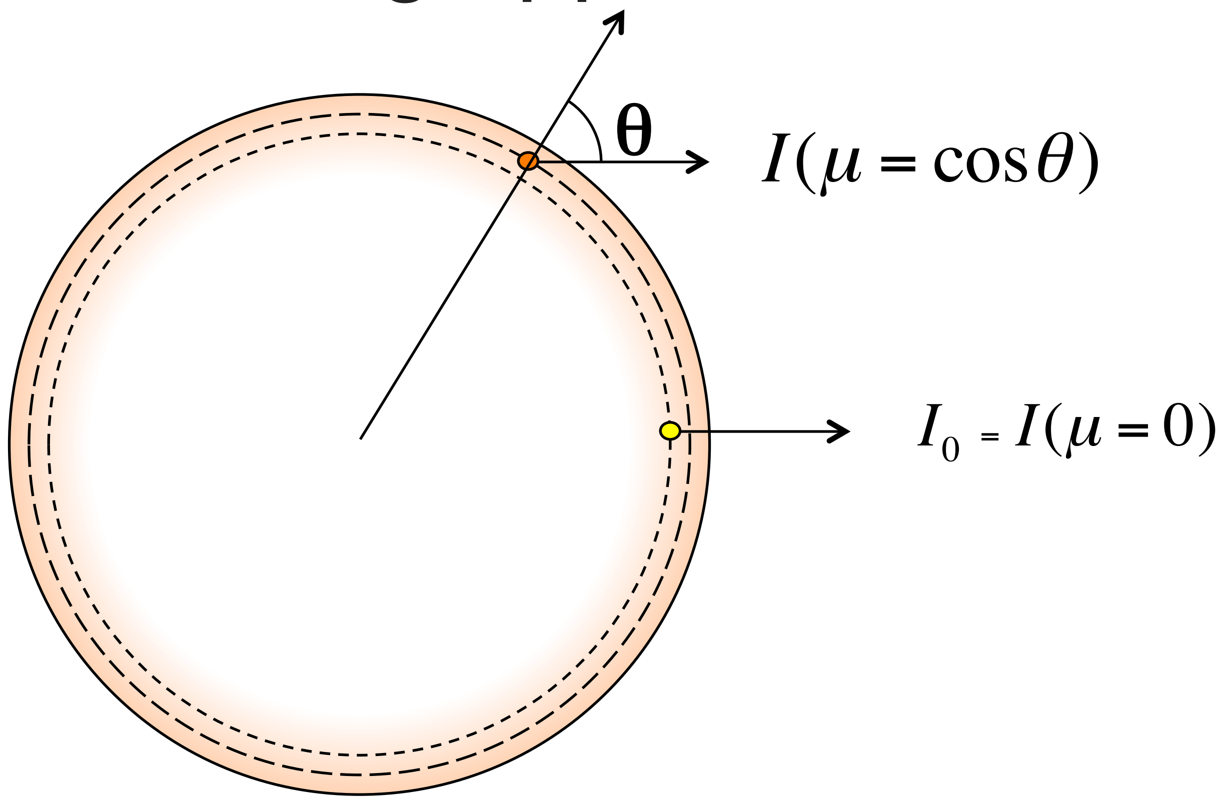
Combining transits with radial-velocity data

Stellar limb darkening



Transit of Venus, 2012

Limb darkening approximations



Linear $$\frac{I(\mu)}{I_0} = 1 - u(1-\mu)$$

Quadratic $$\frac{I(\mu)}{I_0} = 1 - u_1(1-\mu) - u_2(1-\mu^2)$$

Claret 4-parameter $$\frac{I(\mu)}{I_0} = 1 - u_1(1-\mu^{1/2}) - u_2(1-\mu) - u_3(1-\mu^2) - u_4(1-\mu^{3/2})$$

Limb darkening references

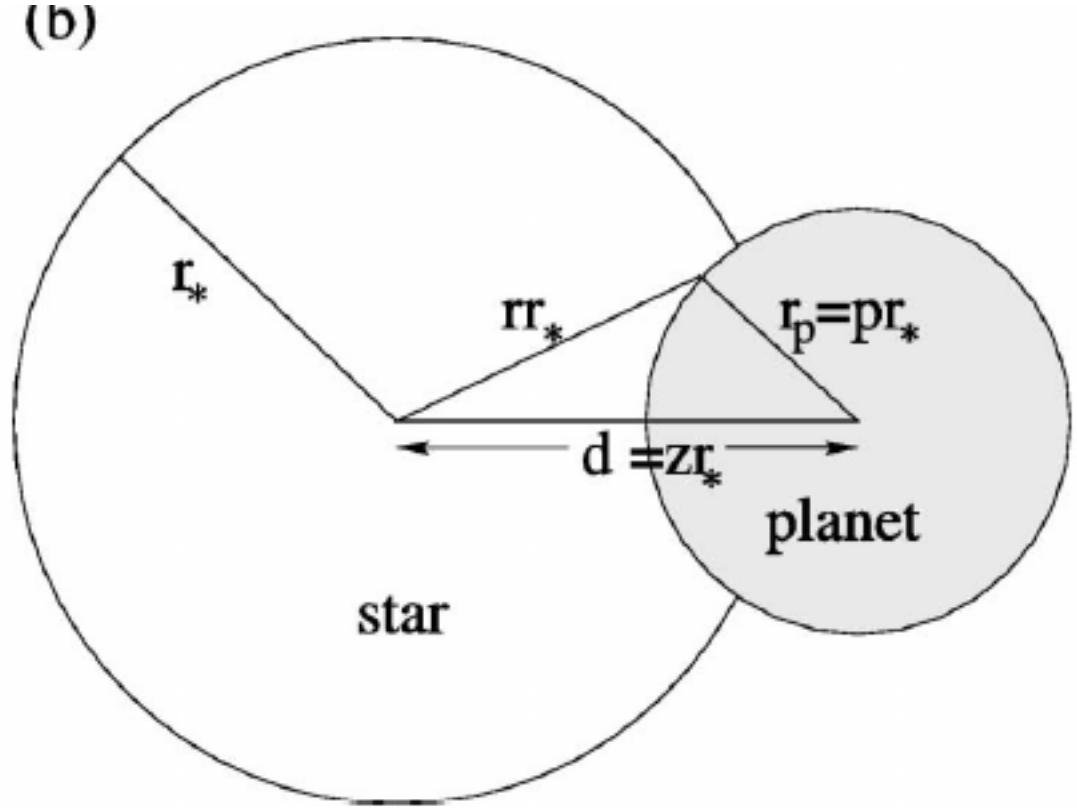
- **http://www.astro.keele.ac.uk/jkt/codes/jktld.html**

Study	Model atmosphere	Teff range	logg range	[M/H] range	Vmicro range
Van Hamme (1993, AJ, 106, 2096)	ATLAS9	3500 to 50000	0.0 to 5.0	0.0	n/a
Díaz-Cordovés et al (1995, A&AS, 110, 329)	ATLAS9	3500 to 50000	0.0 to 5.0	0.0	n/a
Claret et al (1995, A&AS, 114, 247)	ATLAS9	3500 to 50000	0.0 to 5.0	0.0	n/a
Claret (2000, A&A, 363, 1081)	ATLAS9	3500 to 50000	0.0 to 5.0	-5.0 to +0.5	0,1,2,4,8
Claret (2000, A&A, 363, 1081)	Phoenix	2000 to 9800	3.5 to 5.5	0.0	2
Claret & Hauschildt (2003, A&A, 412, 241)	Phoenix	5000 to 10000	0.0 to 5.0	0.0	2
Claret (2004, A&A, 428, 1001)	ATLAS9	3500 to 50000	0.0 to 5.0	-5.0 to +1.0	0,1,2,4,8
Claret (2004, A&A, 428, 1001)	Phoenix	2000 to 9800	3.5 to 5.0	0.0	2
Sing (2010, A&A, 510, A21)	ATLAS9	3500 to 50000	0.0 to 5.0	-5.0 to +1.0	n/a

Study	lin	quad	log	sqrt	exp	3par	4par	Passbands
Van Hamme (1993, AJ, 106, 2096)	✓		✓	✓				bolometric, Strömgren *uvby*, Johnson *UBVRIJHKLMN*, Cousins *RI*
Díaz-Cordovés et al (1995, A&AS, 110, 329)	✓	✓		✓				Strömgren *uvby*, Johnson *UBV*
Claret et al (1995, A&AS, 114, 247)	✓	✓		✓				Cousins *RI*, Teide *JHK*
Claret (2000, A&A, 363, 1081)	✓	✓	✓	✓			✓	bolometric, Strömgren *uvby*, Johnson *UBV*, Cousins *RI*, Teide *JHK*
Claret & Hauschildt (2003, A&A, 412, 241)	✓	✓	✓	✓	✓		✓	Strömgren *uvby*, Johnson *UBV*, Cousins *RI*, Teide *JHK*
Claret (2004, A&A, 428, 1001)	✓	✓	✓	✓			✓	SDSS *ugriz*
Sing (2010, A&A, 510, A21)	✓	✓				✓	✓	CoRoT (white) and Kepler satellite passbands

Transit geometry

- **Key parameters:**
  - **Dimensionless projected separation $z = d/r_*$**
  - **Planet/star radius ratio $p = r_p/r_*$**



Mandel & Agol 2002, ApJ 580,L171

- **Analytic light curve model**
  - **Subroutines in IDL, Fortran, Python are publicly available at:**
  - **http://www.astro.washington.edu/users/agol/transit.html**

- **So all we need to do is**
  - **adopt an appropriate set of limb darkening model coeffs ci**
  - **adopt a value for planet/star radius ratio *p***
  - **compute projected star-planet separation *z*(*t*)**
  - **call *occultsmall* (p, c_1, c_2, c_3, c_4, $z(t)$, $\mu(t)$)**
  - **The output is an array of flux ratios**

$$\mu(t) = \frac{F(t)}{F_0}$$

  - **If working in magnitudes:**

$$\Delta m(t) = -2.5 \log_{10} \mu(t)$$

Example: WASP-19 z-band transit

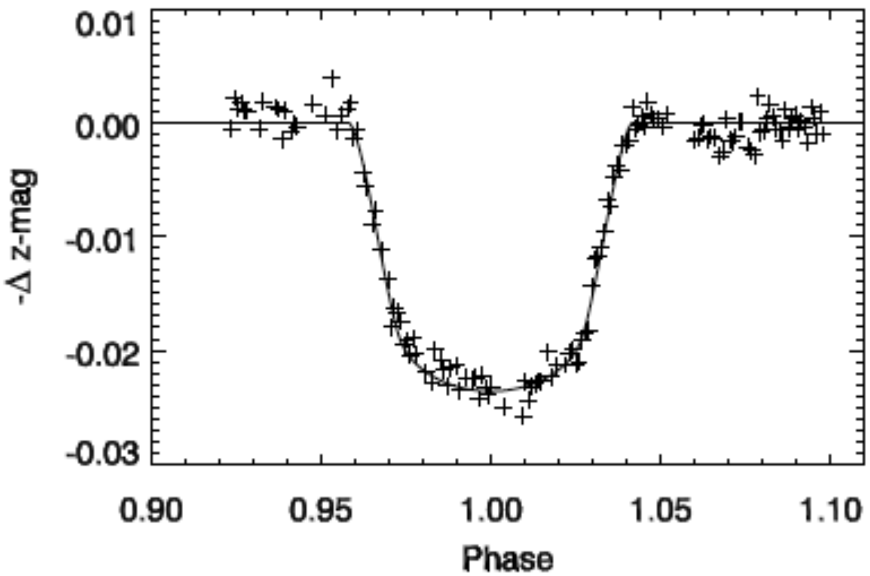


Fig. 2.— FTS z-band photometry of the WASP-19 transit. The data are converted to phase using the ephemeris given in Table 3. Overplotted is the best fitting model transit light curve using the formalism of Mandel & Agol (2002) applying the 4th-order limb darkening coefficients from Claret (2004).

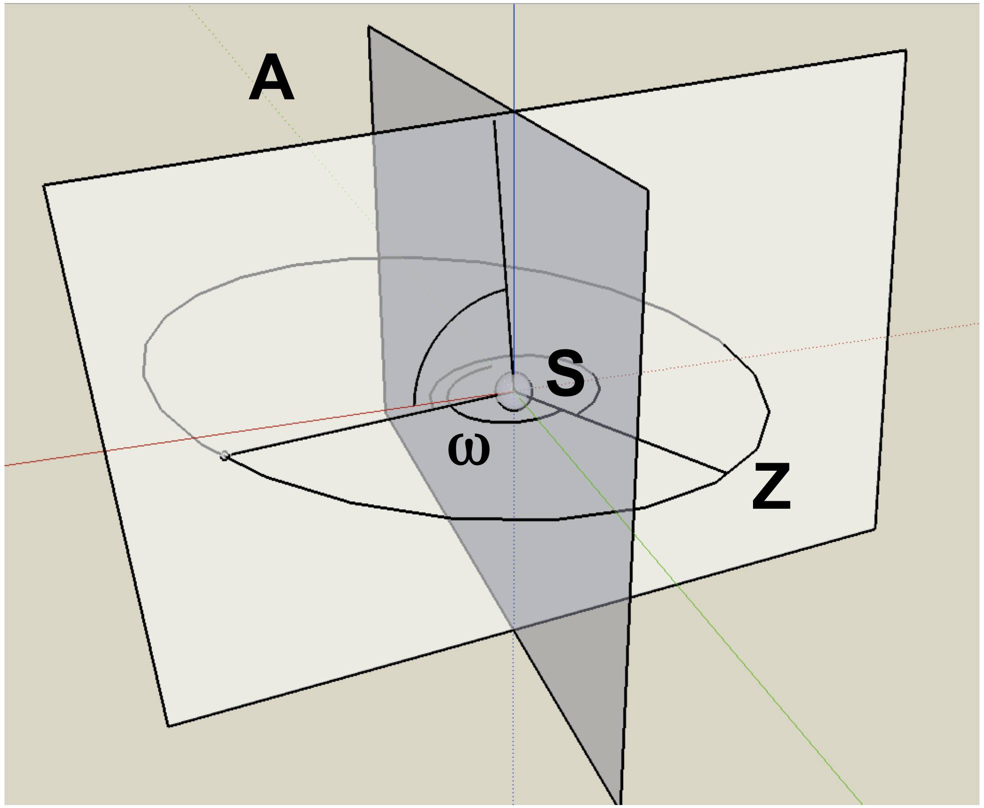
Hebb et al. 2010, ApJ 708, 224

Computing z(t)

- **Orbital elements** t_0, P, e, ω, i
- **Scale parameter** a/R_*
- **Planet radius** R_p/R_*

Argument of periastron, ω

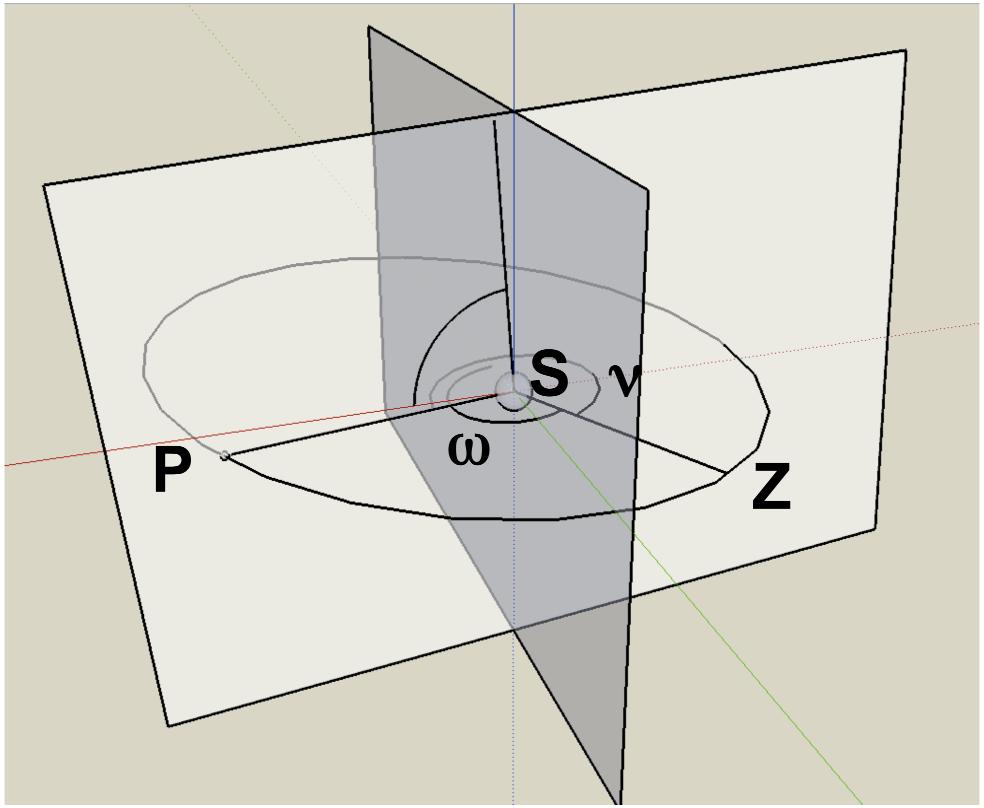
- **Angle ASZ, measured anticlockwise from intersection of orbit plane and sky (A, green axis, planet approaching)…**
- **Around star S in plane of orbit**



… to direction Z of periastron

True anomaly ν of planet

- **Angle ZSP, measured from direction Z of periastron…**
- **Anticlockwise around star S in orbit plane…**
- **To direction P of planet.**



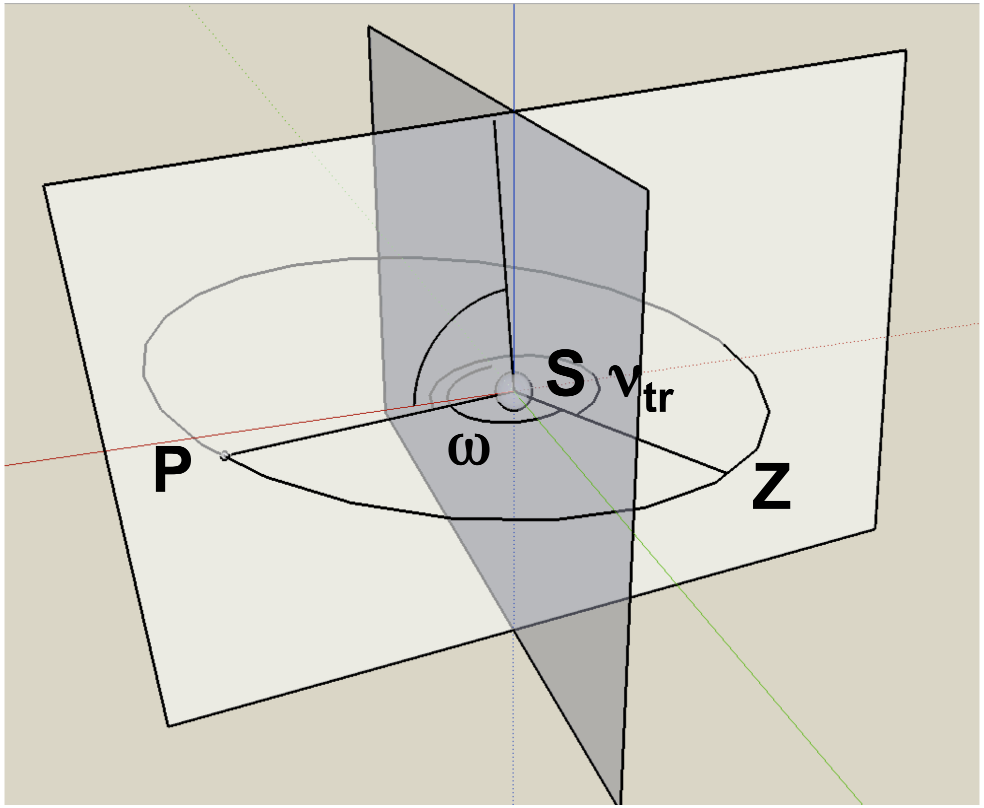
True anomaly at transit

- **True anomaly at primary transit:**

$$\nu_{tr} = \frac{\pi}{2} - \omega$$

- **True anomaly at secondary occultation:**

$$\nu_{occ} = \frac{3\pi}{2} - \omega$$



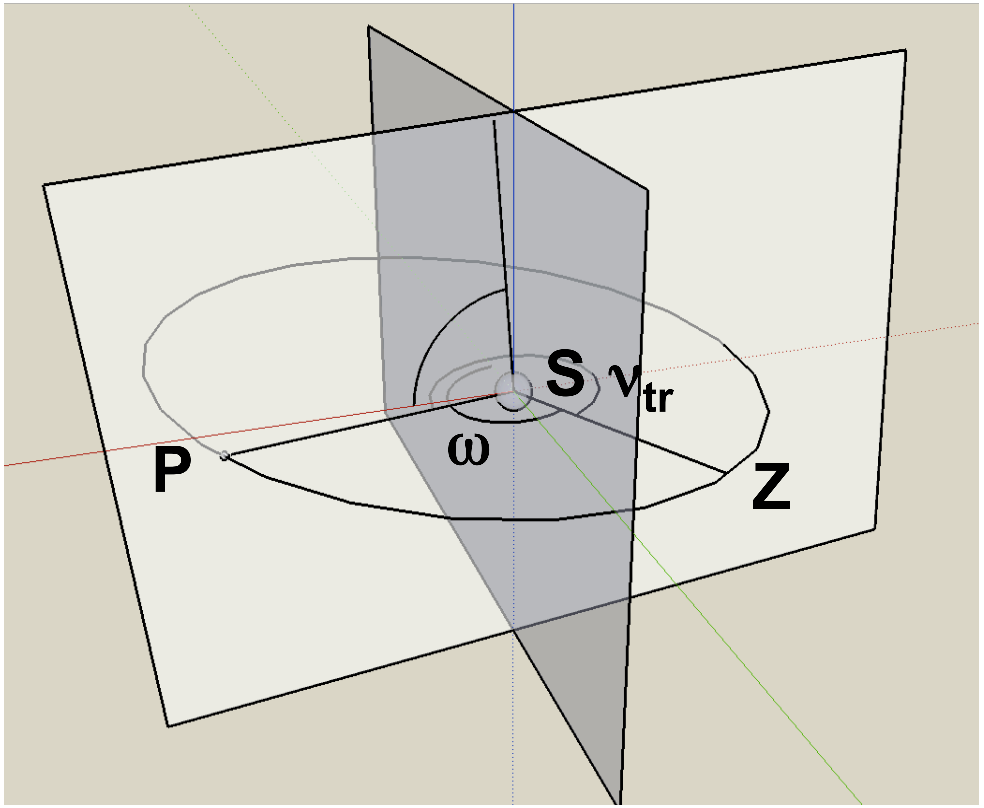
Red axis points to observer

Mean anomaly M

- **Increases with time:**

$$M = \frac{2\pi}{P}(t - t_0)$$

- **P = orbital period**
- **t_0 = time of periastron**



Red axis points to observer

Eccentric anomaly

- **Related to true anomaly ν by:**

$$E = 2\tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\tan\frac{\nu}{2}\right]$$

- **e = orbital eccentricity**

- **Related to mean anomaly by:** $M = E - e\sin E$

Time of periastron

- **True anomaly at transit** $\nu_{tr} = \frac{\pi}{2} - \omega$

- **Mean anomaly at transit:**

$$E_{tr} = 2\tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\tan\frac{\nu_{tr}}{2}\right]$$

- **Time delay from periastron to transit:**

$$\begin{aligned} t_{tr} - t_0 &= \frac{P}{2\pi}M_{tr} \\ &= \frac{P}{2\pi}(E_{tr} - e\sin E_{tr}) \end{aligned}$$

Where is planet at time t?

- **Compute mean anomaly:**

$$M = \frac{2\pi}{P}(t - t_0)$$

- **Guess eccentric anomaly:**

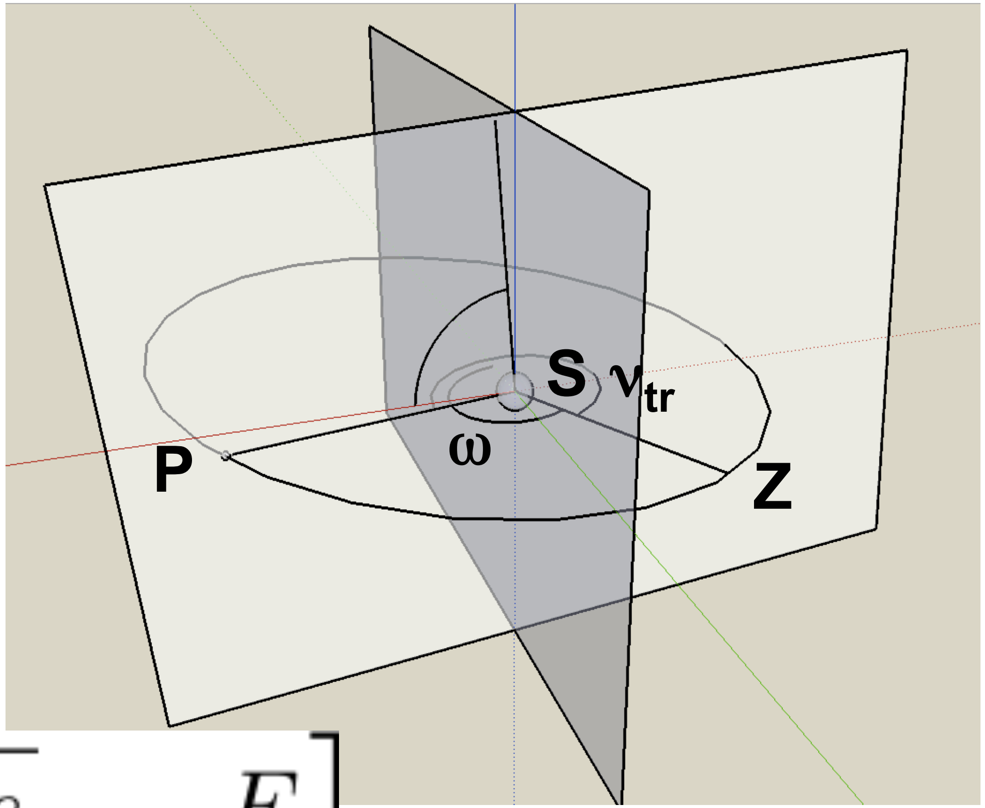
$$E_1 = M$$

- **Iterate:**

$$E_{i+1} = M + e \sin E_i$$

- **On convergence, set:**

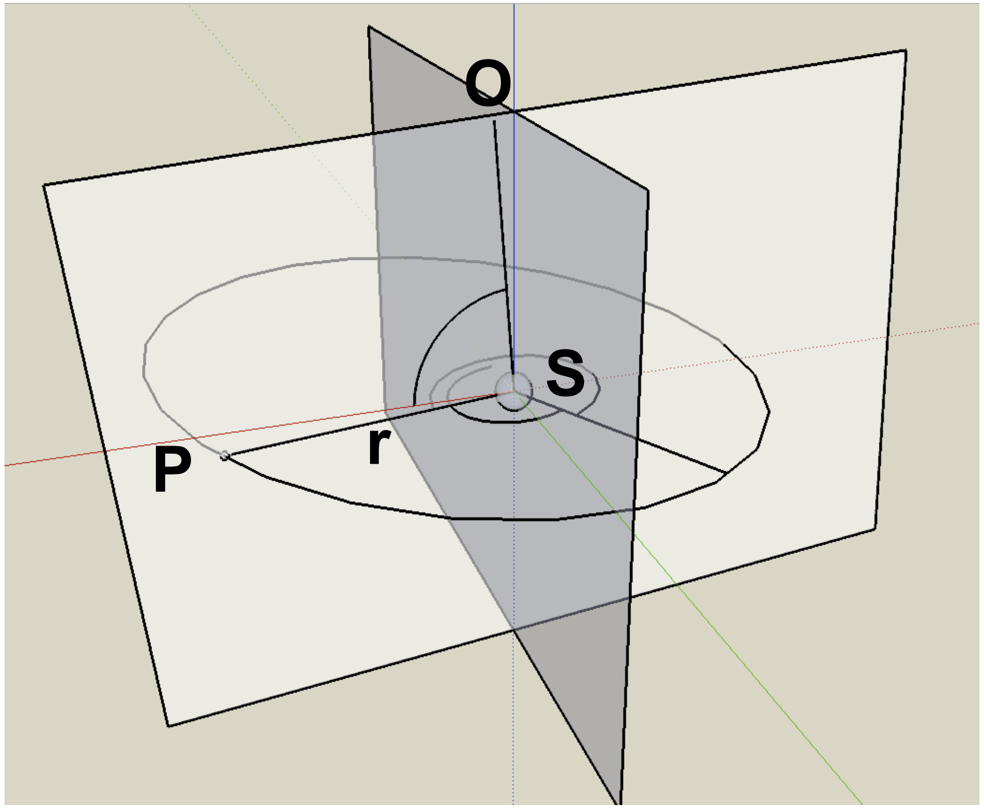
$$\nu = 2 \tan^{-1} \left[\sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2} \right]$$



Star-planet distance, r

- **Instantaneous distance SP**

$$r = a(1 - e\cos E)$$

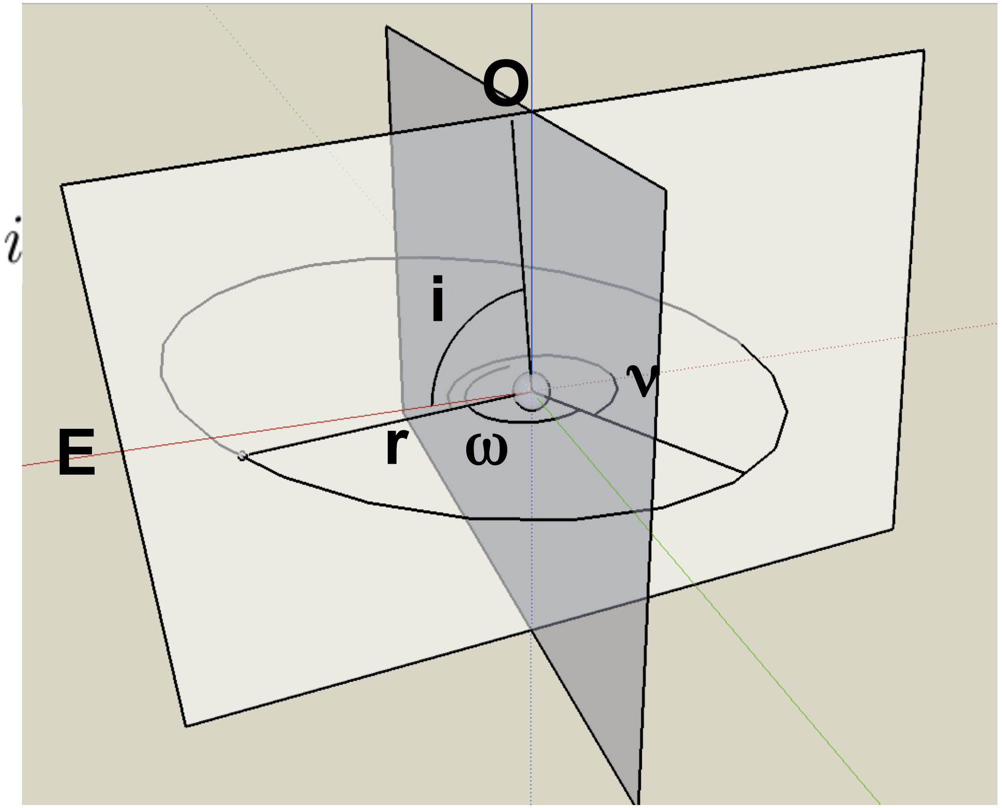


Apparent position on sky

- **As seen from Earth, projected position along x (green), z (blue) axes in plane of sky is:**

$$x_p = r\sin(\nu + \omega - \pi/2)$$

$$z_p = -r\cos(\nu + \omega - \pi/2)\cos i$$



- **Distance toward observer along y (red) axis is:**

$$y_p = r\cos(\nu + \omega - \pi/2)\sin i$$

- **Star-planet-observer phase angle is:**

$$\cos\alpha = y_p/r$$

- **Apparent planet-star separation is:** $r\sin\alpha = \sqrt{x_p^2 + z_p^2}$

Radial velocity

- **Differentiate y_p to get planet's velocity v_p toward observer relative to star**

$$v_p = \frac{dy_p}{d\nu}\frac{d\nu}{dM}\frac{dM}{dt}$$
$$= \frac{2\pi a}{P}\frac{\sin i}{\sqrt{1-e^2}}(e\cos\omega + \cos(\nu+\omega))$$

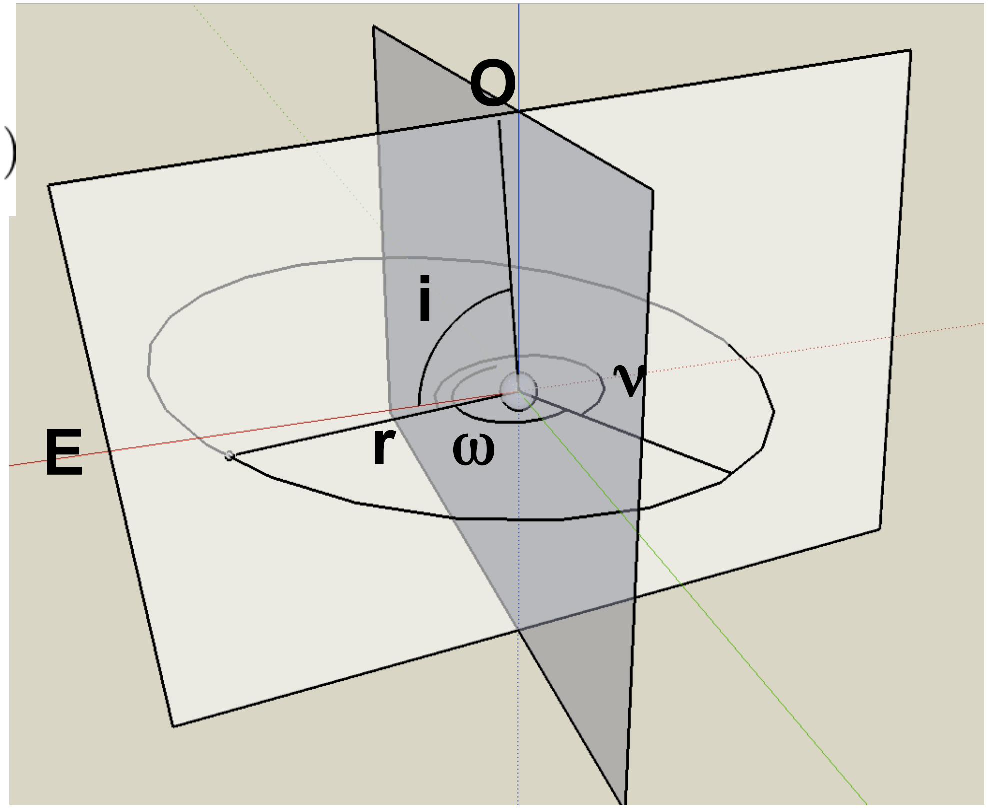
- **Component of star's reflex motion away from observer along red axis :**

$$v_r = K(e\cos\omega + cos(\nu+\omega)) + \gamma$$

- **sin i is implicit in the value of K:**

$$K = \frac{2\pi a}{P}\frac{m_p}{m_* + m_p}\frac{\sin i}{\sqrt{1-e^2}}$$

- **γ is velocity of system centre of mass away from solar system barycentre.**



Transverse velocity

Differentiate xp to get planet's transverse velocity relative to star

$$v_t = \frac{2\pi a}{P} \frac{e \sin\omega + \sin(\nu + \omega)}{\sqrt{1-e^2}}$$

At primary transit, :

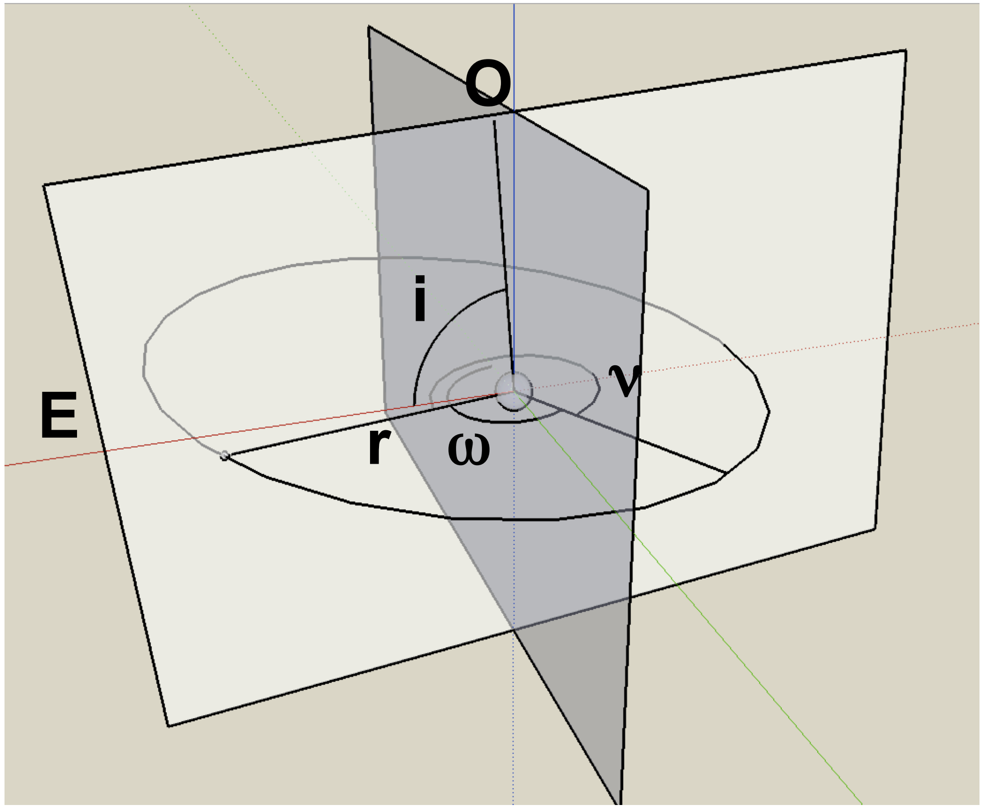
$$\nu_{tr} = \frac{\pi}{2} - \omega$$

$$\Rightarrow v_t = \frac{2\pi a}{P} \frac{e \sin\omega + 1}{\sqrt{1-e^2}}$$

At secondary eclipse:

$$\nu_{occ} = \frac{3\pi}{2} - \omega$$

$$\Rightarrow v_t = \frac{2\pi a}{P} \frac{e \sin\omega - 1}{\sqrt{1-e^2}}$$



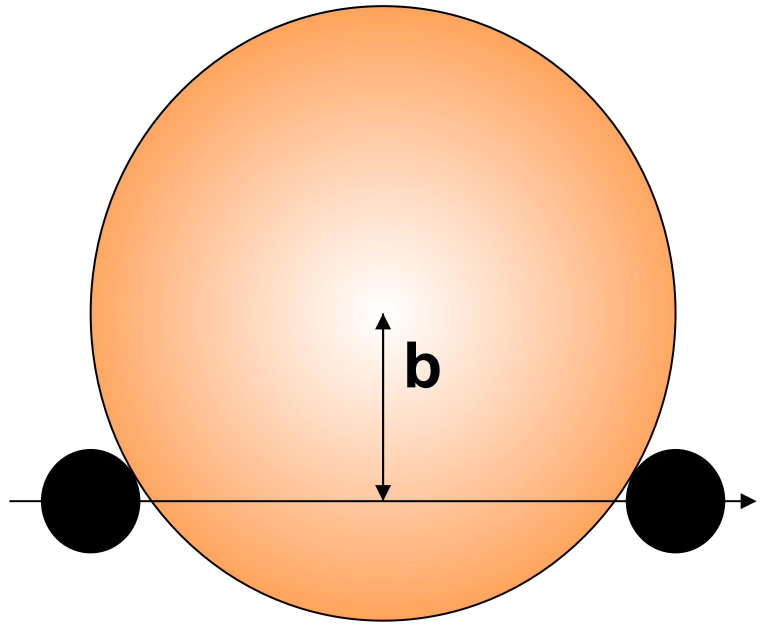
Primary transit duration

- **Separation of centres in units of stellar radius R_s**

$$h = 1 + R_p/R_s$$

- **Impact parameter at primary transit:**

$$b = \frac{z_p}{R_s} = -\frac{a}{R_s}\frac{1-e^2}{1+e\sin\omega}\cos i$$



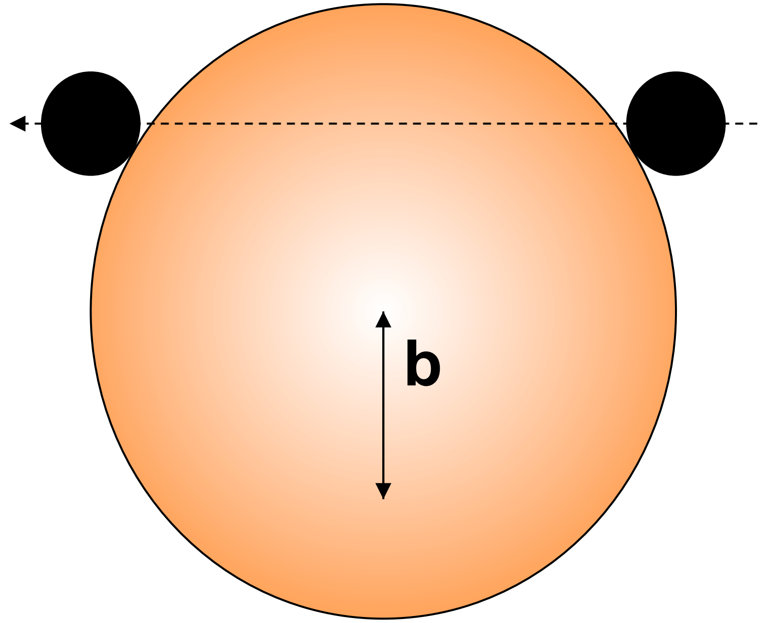
- **Approximate transit duration:**

$$\frac{t_{tr}}{P} \simeq \frac{R_s}{a}\frac{\sqrt{(1+R_p/R_s)^2-b^2}}{\pi}\frac{\sqrt{1-e^2}}{1+e\sin\omega}$$

Secondary eclipse duration

- **Impact parameter at secondary eclipse:**

$$b_{occ} = \frac{a}{R_s}\frac{1-e^2}{1-e\sin\omega}\cos i$$
$$= b\frac{1+e\sin\omega}{1-e\sin\omega}$$



- **Approximate eclipse duration:**

$$\frac{t_{occ}}{P} = \frac{R_s}{a}\frac{\sqrt{(1+R_p/R_s)^2-b_{occ}^2}}{\pi}\frac{\sqrt{1-e^2}}{1-e\sin\omega}$$
$$= \frac{t_{tr}}{P}\frac{\sqrt{(1+R_p/R_s)^2-b_{occ}^2}}{\sqrt{(1+R_p/R_s)^2-b^2}}\frac{1+e\sin\omega}{1-e\sin\omega}$$

Ingress and egress duration

- **Separation of centres at first contact**

$$h = 1 + R_p/R_s$$

- **Separation of centres at second contact:**

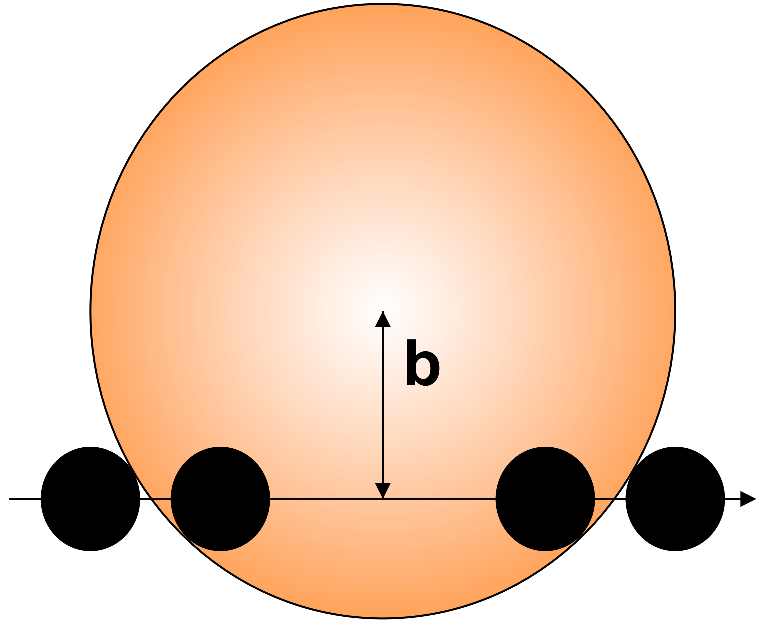
$$h = 1 - R_p/R_s$$

- **Approximate ingress/egress duration at primary transit:**

$$\frac{t_{12}}{P} \simeq \frac{t_{tr}}{2P}\left(1 - \frac{\sqrt{(1-R_p/R_s)^2 - b^2}}{\sqrt{(1+R_p/R_s)^2 - b^2}}\right) \simeq \frac{t_{tr}}{P}\frac{R_p/R_s}{1-b^2}$$

- **And at secondary eclipse:**

$$\frac{t_{12}}{P} \simeq \frac{t_{occ}}{2P}\left(1 - \frac{\sqrt{(1-R_p/R_s)^2 - b_{occ}^2}}{\sqrt{(1+R_p/R_s)^2 - b_{occ}^2}}\right) \simeq \frac{t_{occ}}{P}\frac{R_p/R_s}{1-b_{occ}^2}$$



Markov Chain Monte Carlo (MCMC)

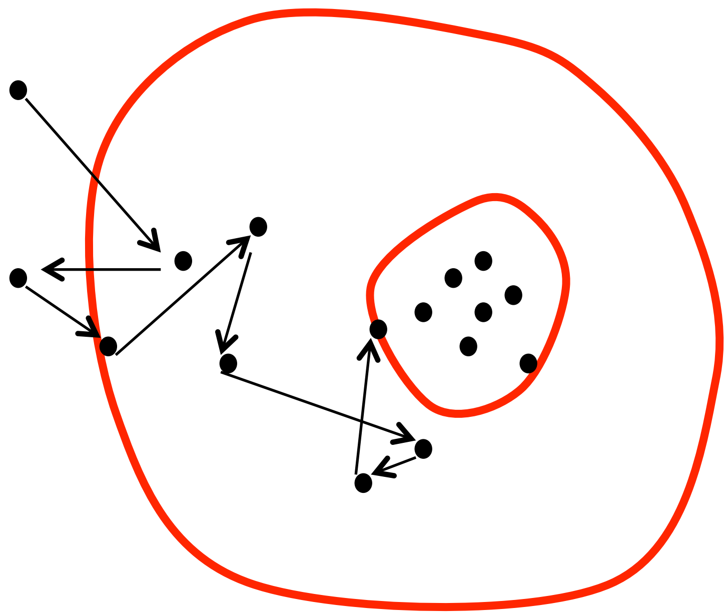
1. Start with M+1 points in the M-dimensional parameter space.
2. Evaluate σ_i for each parameter (and covariance matrix) from last n points.
3. Take a **random step**, e.g. using a Gaussian random number with same σ_i (and covariances) as recent points.

$$\Delta\alpha_i \sim G(0,\sigma_i^{\,2})$$

4. Evaluate $\Delta\chi^2 = \chi^2_{new} - \chi^2_{old}$ and keep the step with probability

$$P = \min\left[\,1,\ \exp\left(-\Delta\chi^2/2\right)\right]$$

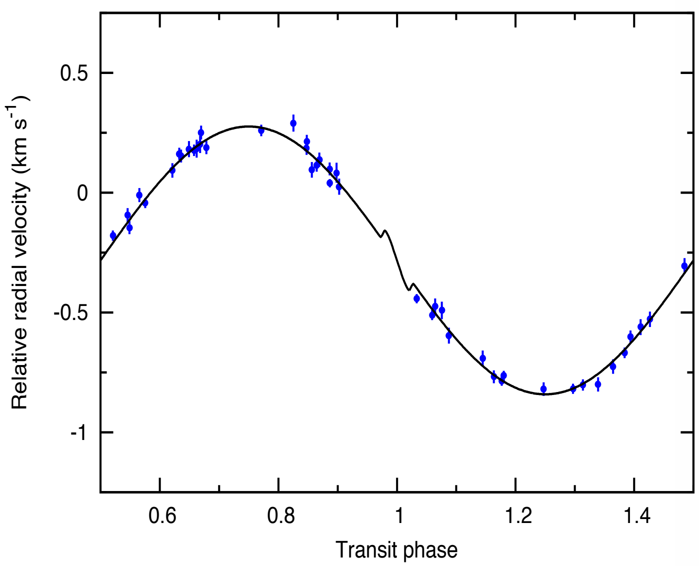
5. Iterate steps 2-4 until "convergence".

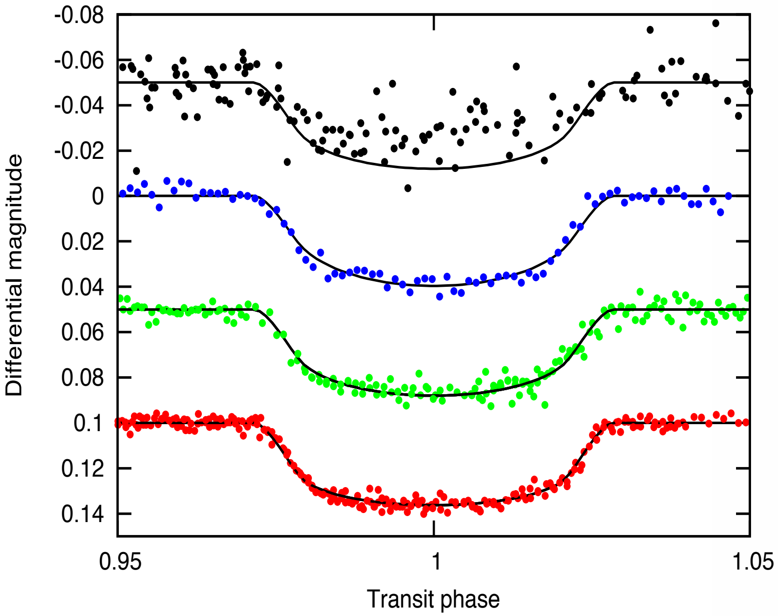


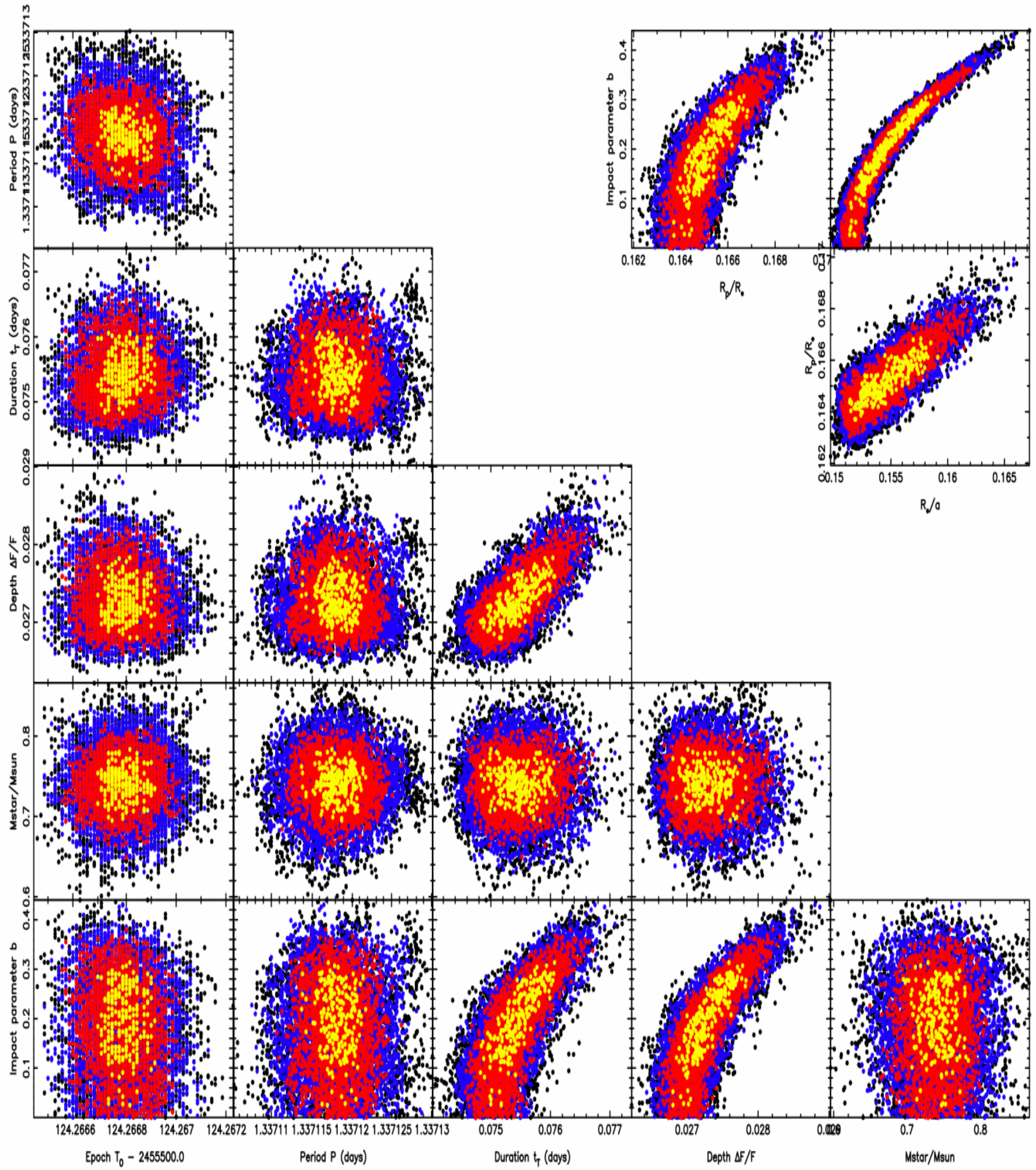
MCMC requires no derivatives :)

MCMC generates a "chain" of points tending to move downhill, then settling into a pattern matching the **posterior distribution** of the parameters. Can escape from local minima.
Can also include prior distributions on the parameters.

Example: MCMC fit of exoplanet model to transit lightcurves and radial velocity curve data.







Vietri Sul Mare 2015

Summary

- **Use BLS to determine approximate**
  - **orbital period**
  - **epoch of mid-transit**
  - **transit duration**
  - **transit depth**
- **Use dynamical model to determine planet-star separation as a function of time.**
- **Use full transit model with appropriate limb darkening to determine flux deficit and/or RV at each time of observation.**
- **Measure likelihood of data given model and parameter values.**
- **Use MCMC to determine most likely parameters and posterior probability distributions.**