Lecture 1: Introduction to exoplanetary transits

Transit probability

Early detections

Transit depths and durations

Model-independent system parameters

Transiting planets

- **Struve 1952: Observatory 72, 199.**

PROPOSAL FOR A PROJECT OF HIGH-PRECISION STELLAR RADIAL VELOCITY WORK

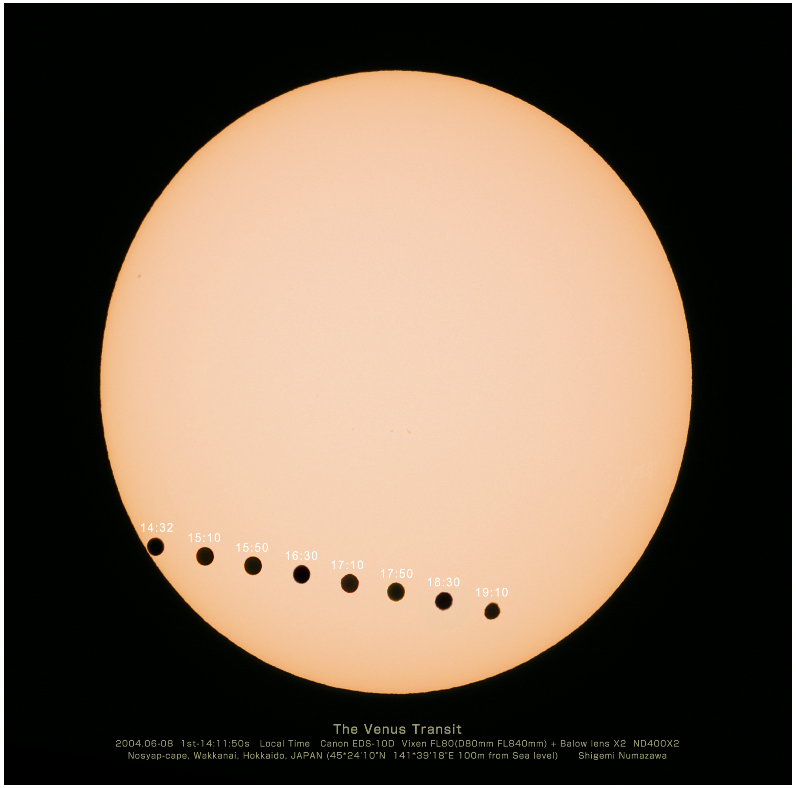
By Otto Struve

But there seems to be no compelling reason why the hypothetical stellar planets should not, in some instances, be much closer to their parent stars than is the case in the solar system. It would be of interest to test whether there are any such objects.

There would, of course, also be eclipses. Assuming that the mean density of the planet is five times that of the star (which may be optimistic for such a large planet) the projected eclipsed area is about 1/50th of that of the star, and the loss of light in stellar magnitudes is about 0·02. This, too, should be ascertainable by modern photoelectric methods, though the spectrographic test would probably be more accurate. The advantage of the photometric procedure would be its fainter limiting magnitude compared to that of the high-dispersion spectrographic technique.

Transits

Simplest method: look for drop in stellar flux due to a planet transiting across the stellar disc



Venus Transit in 2004

International Space Station and Space Shuttle crossing the disk of the Sun

Transits only occur if the orbit is almost edge-on. What is the probability?

Orbital inclination, i

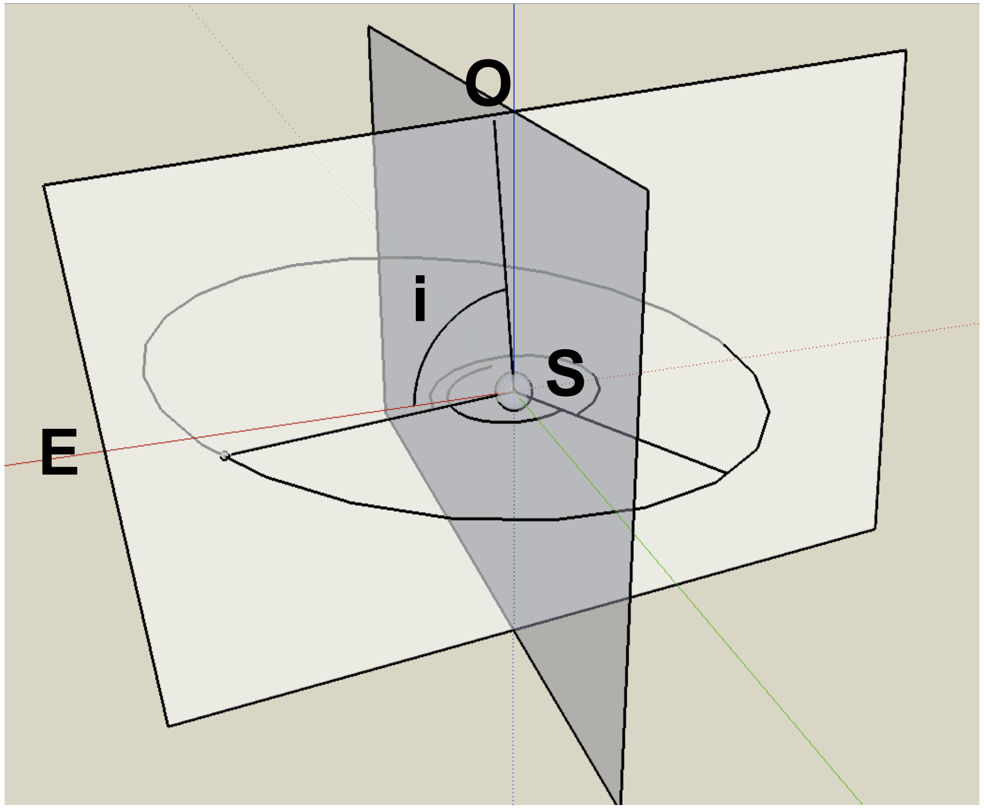
Angle OSE, subtended at star S

by direction O of orbital pole

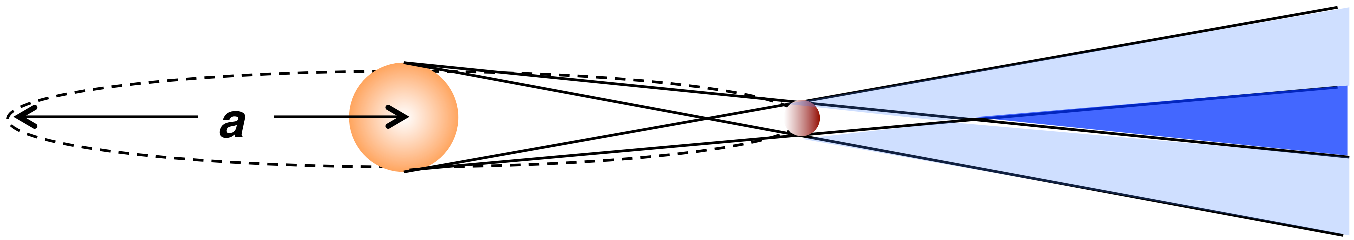
and direction E to Earth (along red axis)

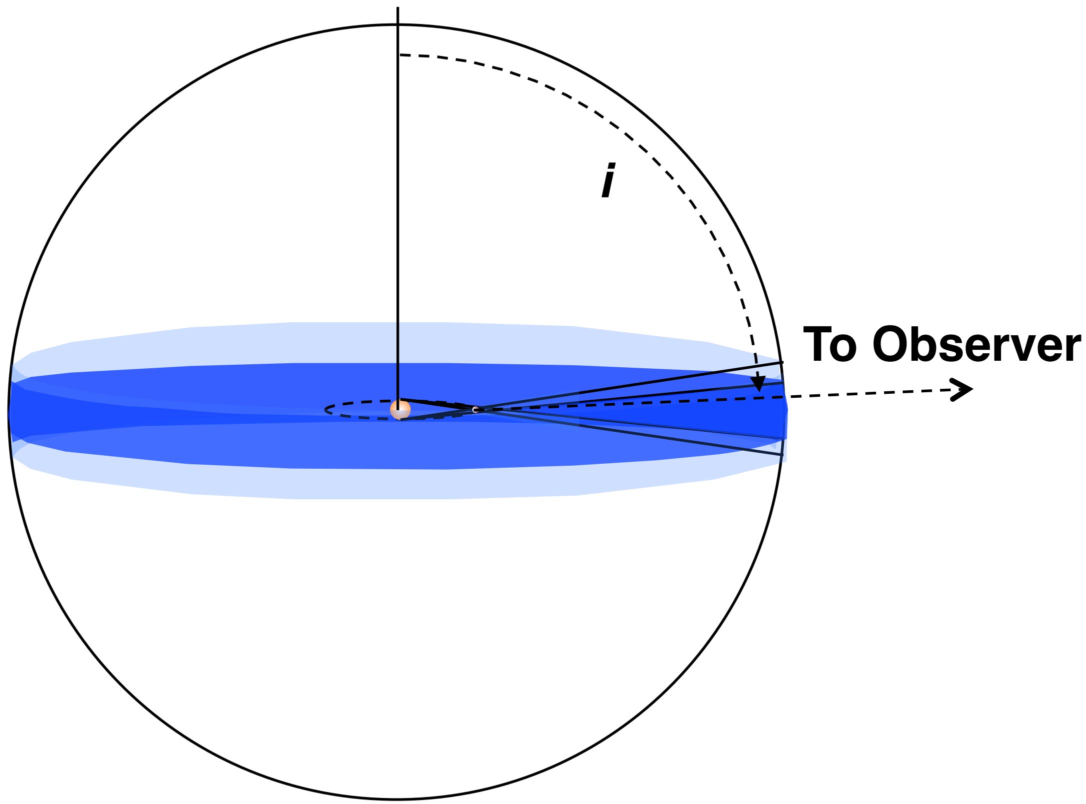
i = 90° => equator-on

i = 0° => pole-on



Full and grazing transits





- **Grazing transit**

$$a\cos i \leq R_* + R_p$$

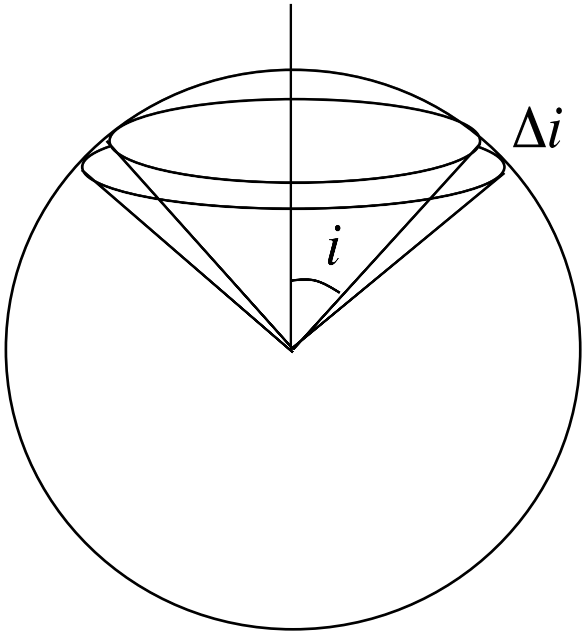
- **Full transit**

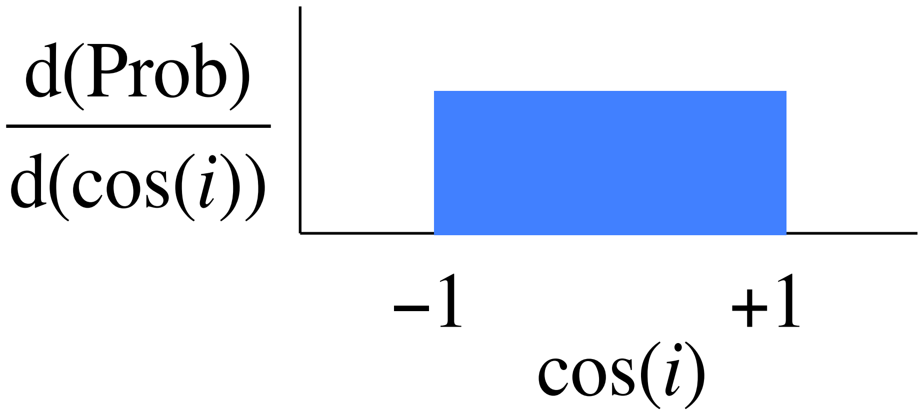
$$a\cos i \leq R_* - R_p$$

Random Orbit Orientation

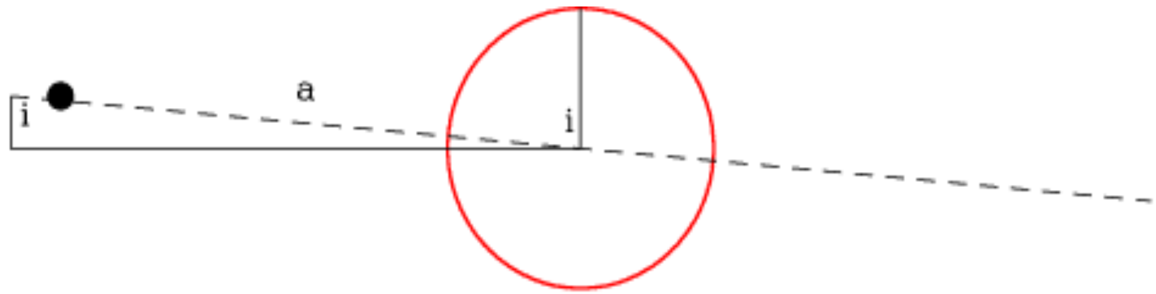
- **Probability that angle between pole of orbit and line of sight lies between i and i + di?**

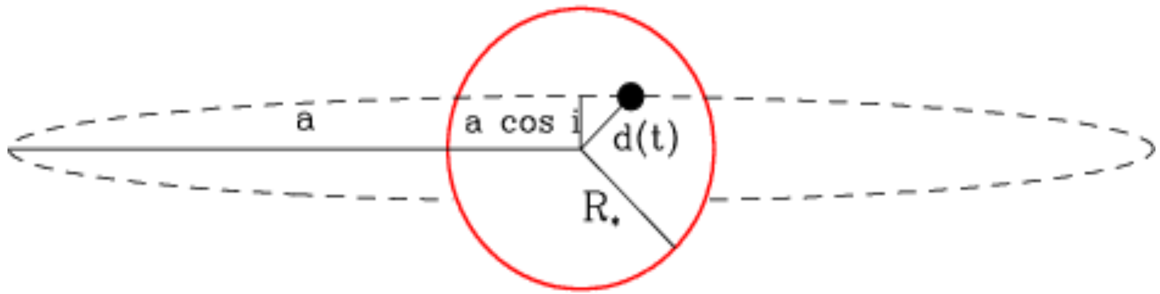
$$d(\mathrm{Prob}) = \frac{\mathrm{d}\Omega}{4\pi} = \frac{2\pi\,\sin(i)\,\mathrm{d}(i)}{4\pi} = \frac{\mathrm{d}(\cos(i))}{2}$$





Transit Probability





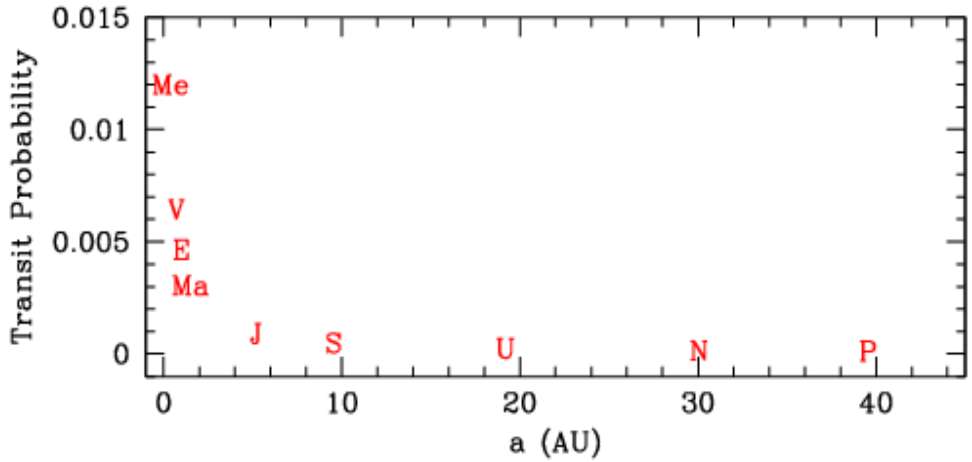
Transits occur only in nearly edge-on orbits: $$a \cos i \le R_* + R_p$$

Random orbit orientation -> probability uniform in cos(*i*).

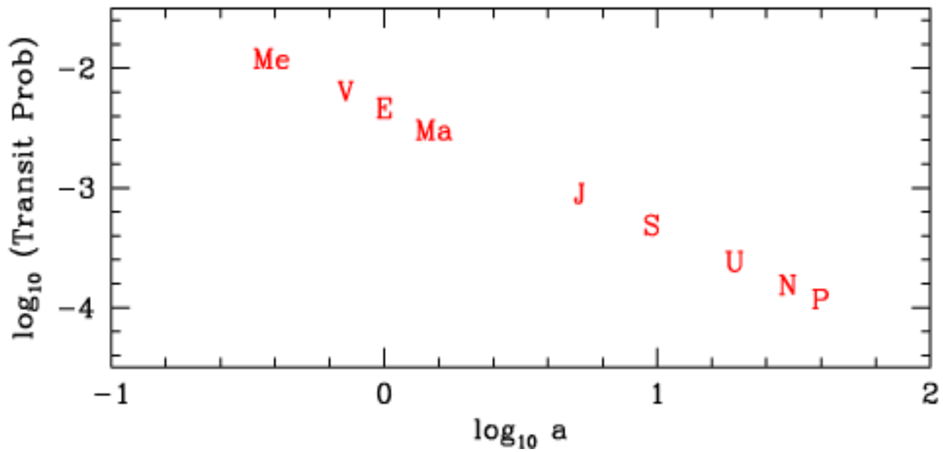
Transit probability is then: $$\mathrm{Prob}\left(\cos \mathrm{i} < \frac{R_* + R_p}{a}\right) = \frac{R_* + R_p}{a} \approx \frac{R_*}{a}$$

Transit surveys find planets in small orbits around large parent stars.

Transit Probability

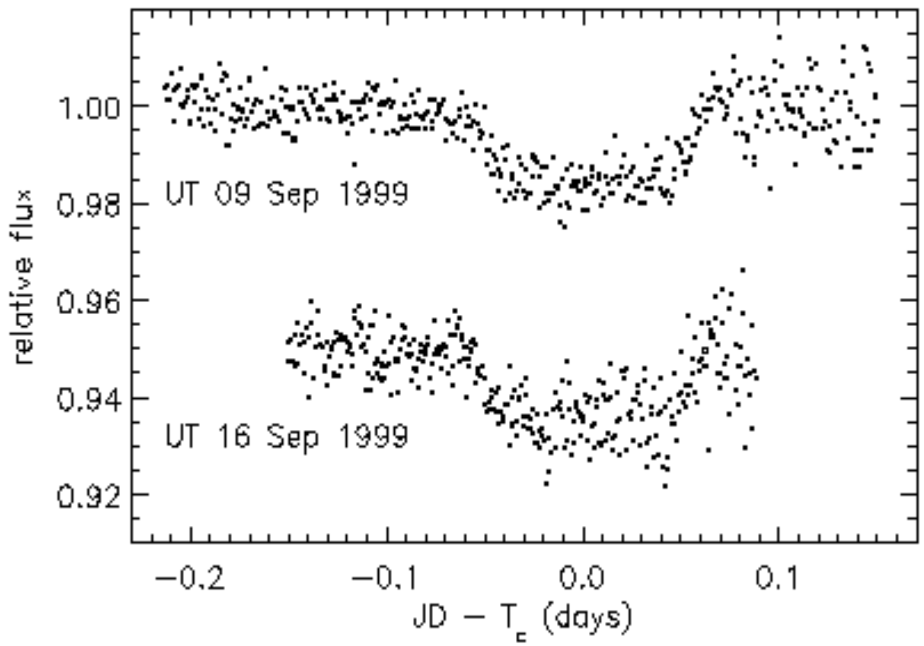


$$\text{Prob} \approx \frac{R_*}{a} \approx 0.005\left(\frac{R_*}{R_{sun}}\right)\left(\frac{1AU}{a}\right)$$



- Hot planets more likely to be detected.
- Prob = 0.5 % at 1 AU, Prob = 0.1 % at 5 AU (Jupiter's orbit)
- Prob = 10% at 0.05 AU (Hot Jupiters)
- Thousands of stars must be monitored to discover planets by spotting their transits.

1999 *First Transiting Exoplanet*



HD 209458 V=7.6 mag

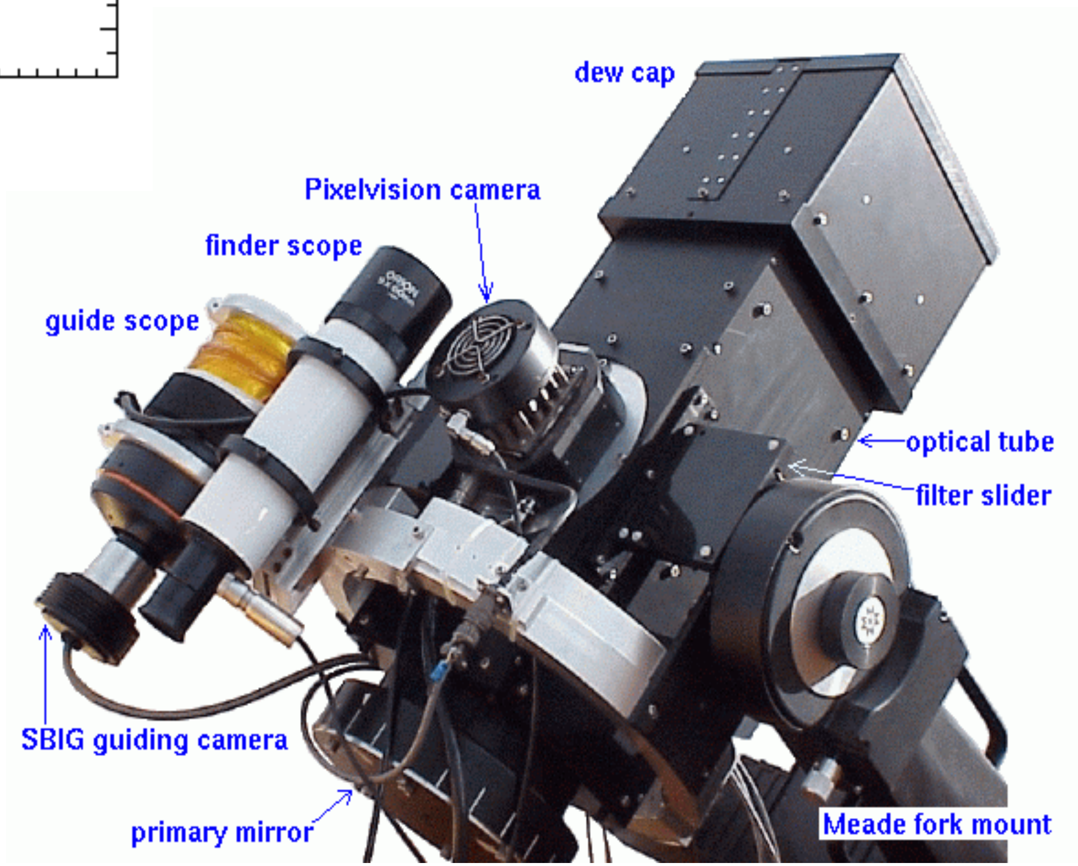
1.6% deep "winks"

last 3 hours

repeat every 3.5 days

Charbonneau & Brown (2000)

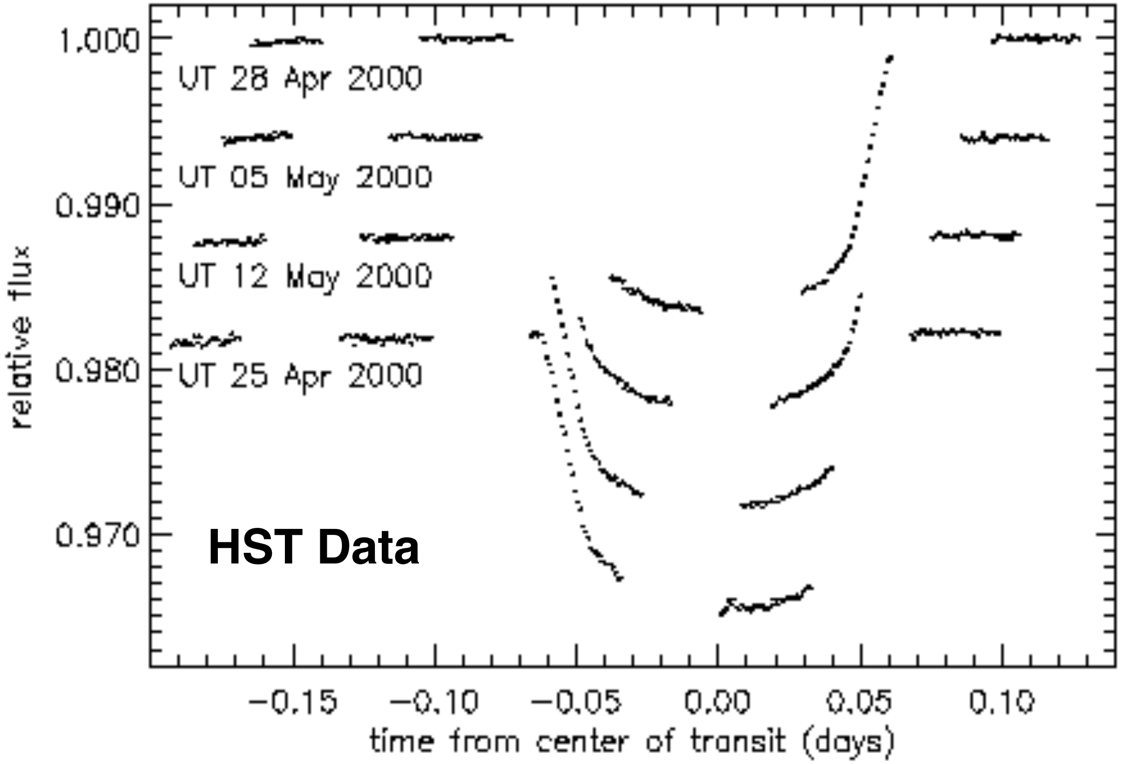
STARE **10 cm telescope**



A Very Big Discovery by a grad student using a Very Small Telescope!

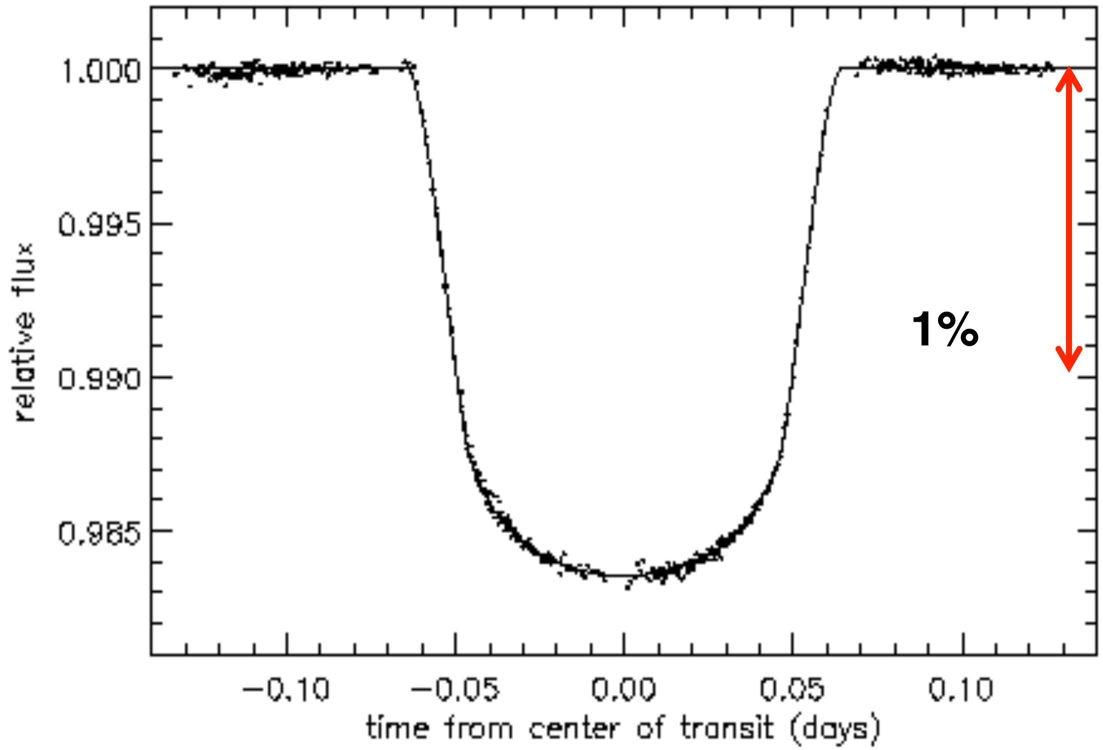
HD 209458 Transits HST/STIS

Brown et al. (2001)

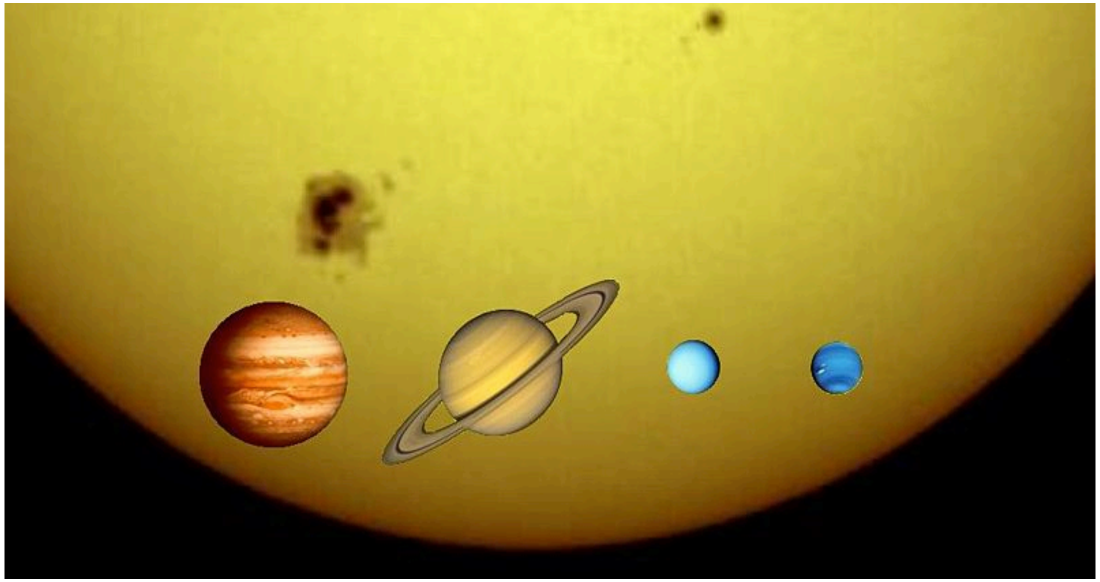


P = 3.52 d
a = 0.046 AU
m_V = 7.8
$\Delta f / f$ = 0.017 mag (1.6%)
i = 86°.6 ± 0°.2
r_p = 1.35 ± 0.06 r_J

From radial velocities
$m \sin i$ = 0.69 m_J
⇒ "bloated" gas giant



Transit Depth



What fraction of the star's disk does the planet cover?

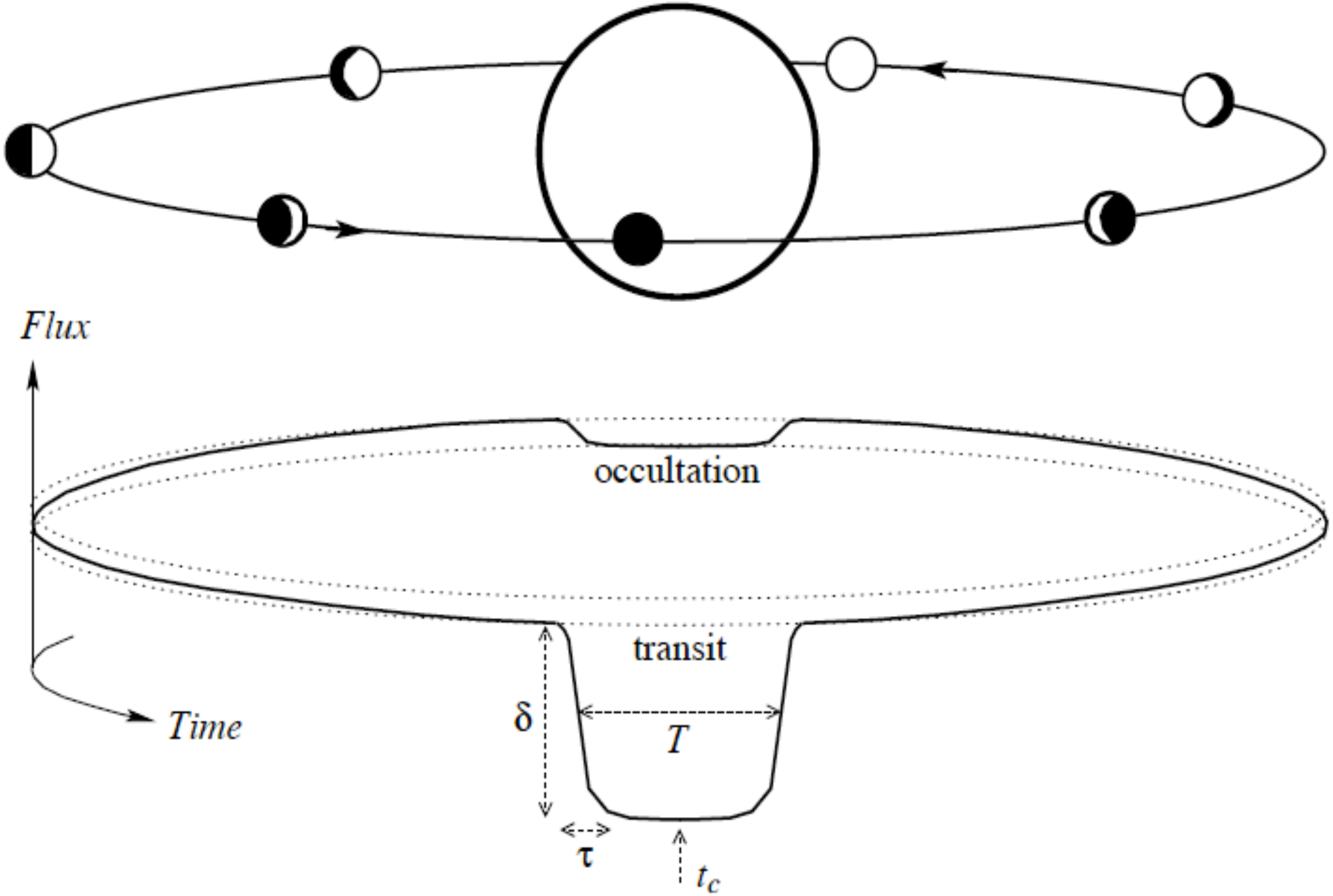
$$\frac{\Delta f}{f} \approx \left(\frac{r_p}{R_*}\right)^2 = 0.01\left(\frac{r_p}{r_{Jup}}\right)^2\left(\frac{R_*}{R_{sun}}\right)^{-2}$$

Find star radius from its spectral type.

Observed depth yields planet/star area ratio.

Transit duration

- **Winn 2008, IAU Symp. 253**



Transit duration and stellar density

- **Seager & Mallén-Ornelas, 2003, ApJ 585, 1038**
- **Simplest case: circular orbit, i=90 degrees**
  - **Relative transit duration:**

$$\frac{T}{P} \approx \frac{2R_*}{2\pi a}$$

  - **Kepler's 3rd Law:**

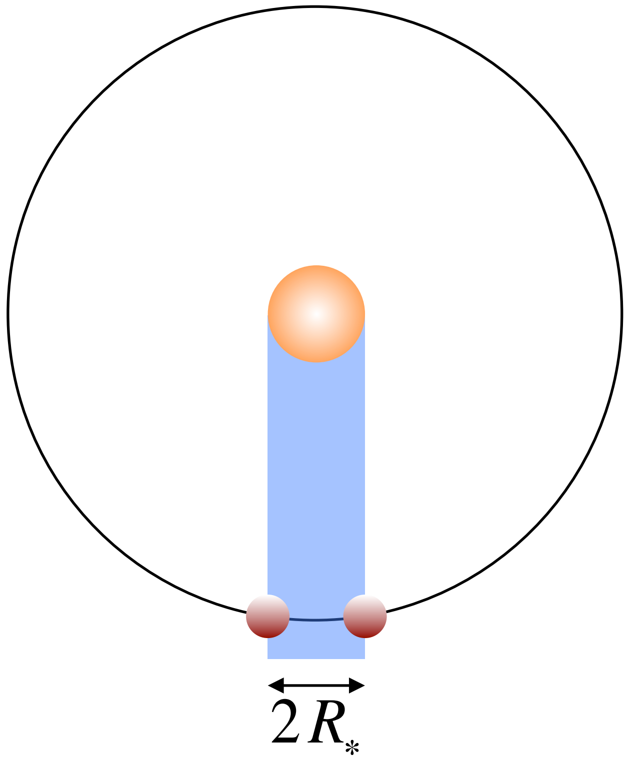
$$a^3 = GM_*\left(\frac{P}{2\pi}\right)^2$$

  - **Hence**

$$\frac{T}{P} \approx \frac{R_*}{\pi a} = \frac{R_*}{\pi}\left(\frac{4\pi^2}{GM_*P^2}\right)^{1/3}$$

$$T \approx 3\mathrm{h}\left(\frac{P}{4\mathrm{d}}\right)^{1/3}\left(\frac{\rho_*}{\rho_{Sun}}\right)^{-1/3}$$

circumference = $2\pi a$



Planetary surface gravity

- **Southworth, Wheatley & Sams 2007, MNRAS 379, L11**
- **Simplest case: circular orbit, i=90 degrees**
  - **Stellar orbital acceleration:**

$$\frac{dv_r}{dt} = \frac{2\pi K}{P} = \frac{GM_p}{a^2} = g_p \frac{R_p^2}{a^2}$$

  - **Inverse square law of gravitation:**

$$\frac{2\pi K}{P} = g_p \left(\frac{R_p}{R_*}\right)^2 \left(\frac{R_*}{a}\right)^2$$

Stellar radial acceleration at conjunction

Planet surface gravity

Transit depth

Transit duration

Planetary density

- **Need to know planet radius and surface gravity:**

$$g_p = \frac{GM_p}{R_p^2} = GR_p\rho_p = GR_*\left(\frac{R_p}{R_*}\right)\rho_p$$

- **Use stellar angular diameter θ and parallax π:**

$$R_* = \theta d = \frac{\theta}{\pi}$$

- **Hence get planetary bulk density:**

$$\rho_p = \frac{g_p \pi}{G\theta}\left(\frac{R_*}{R_p}\right)$$

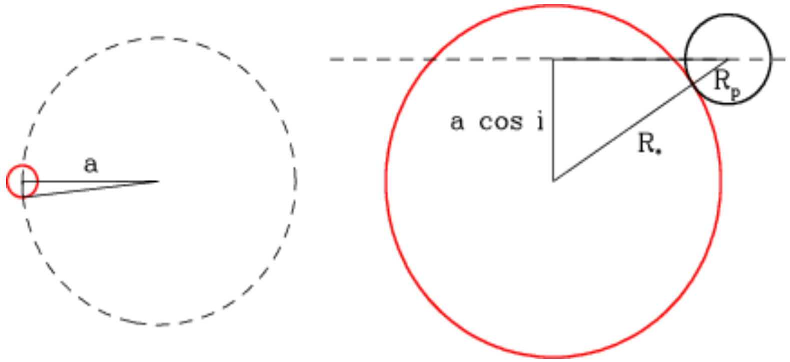
Stellar parallax

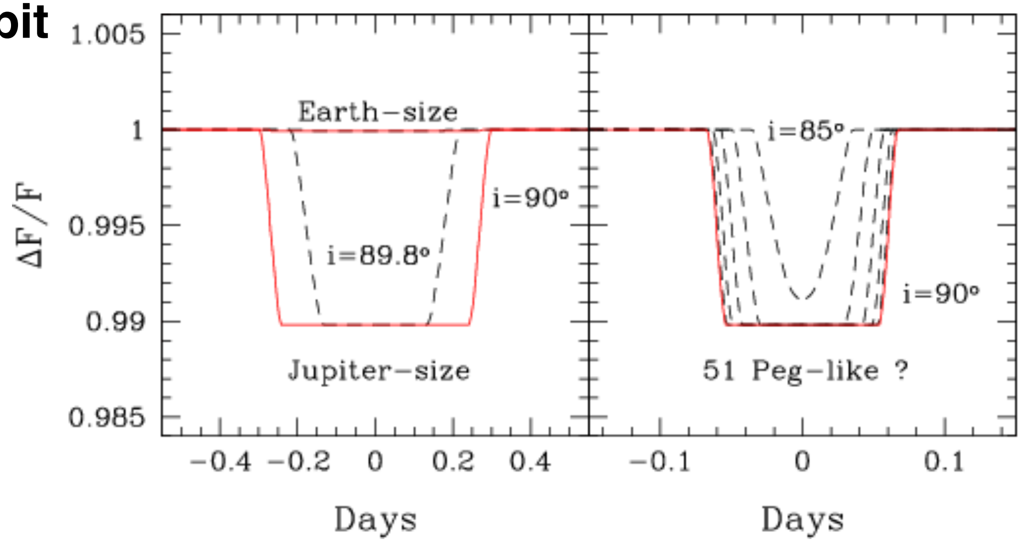
Stellar angular diameter

Transit depth

Transit Duration ($i < 90^o$)

Transit duration reduces to 0 as orbit tips away from edge-on.





Time from first to last contact:

$$t_T = \frac{P}{\pi} \arcsin\left(\frac{R_*}{a} \left\{ \frac{\left[1 + (R_p/R_*)\right]^2 - \left[(a/R_*)\cos i\right]^2}{1 - \cos^2 i} \right\}^{1/2} \right);$$

For cos i << 1 this becomes:

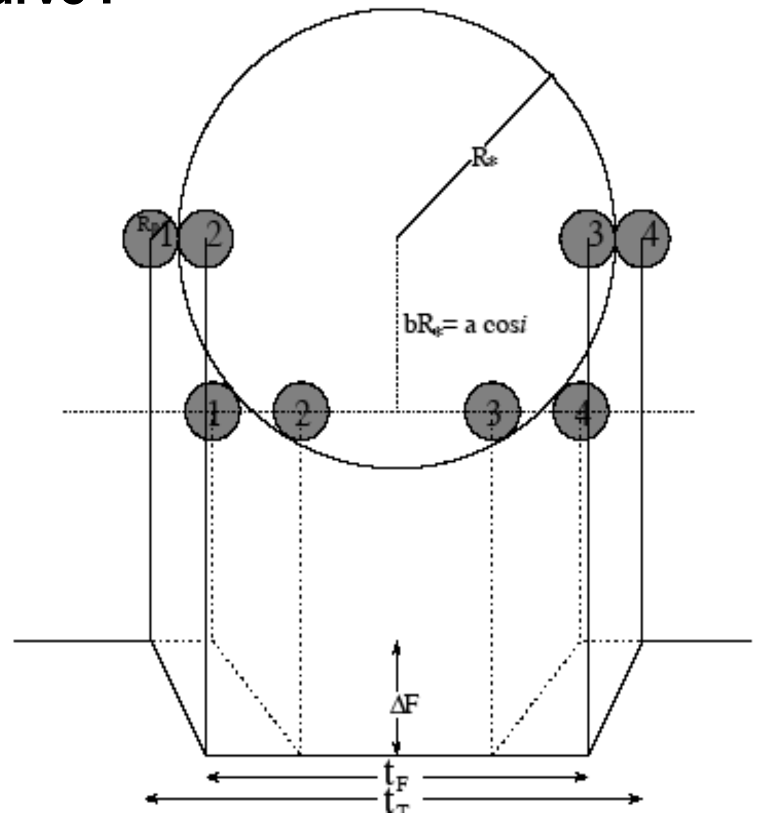
$$t_T = \frac{P R_*}{\pi a} \sqrt{\left(1 + \frac{R_p}{R_*}\right)^2 - \left(\frac{a}{R_*}\cos i\right)^2}.$$

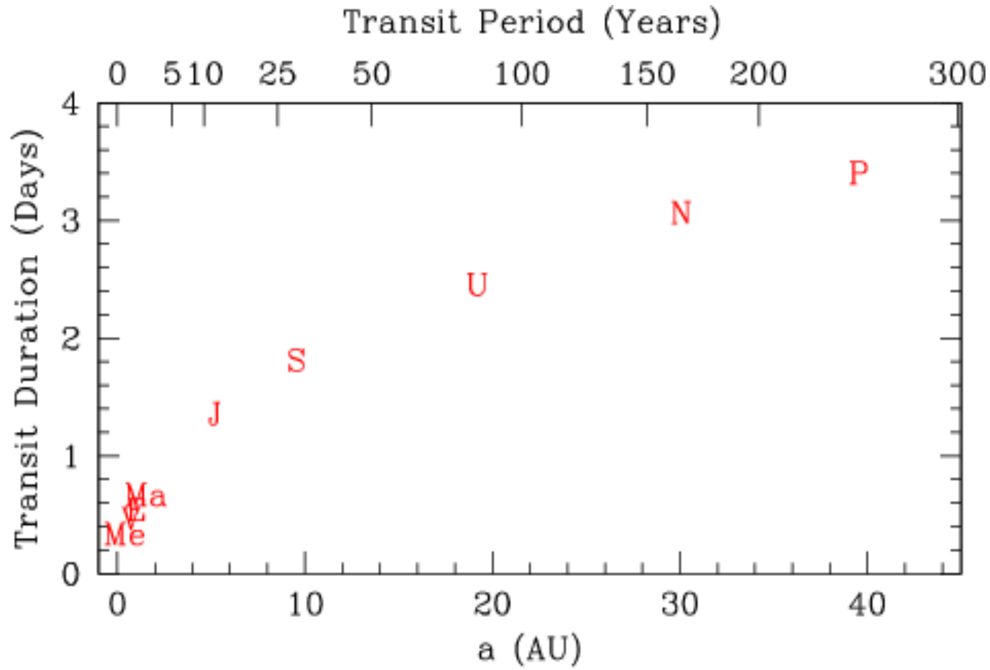
Seager & Mallen-Ornelas 2003, ApJ 585, 1038

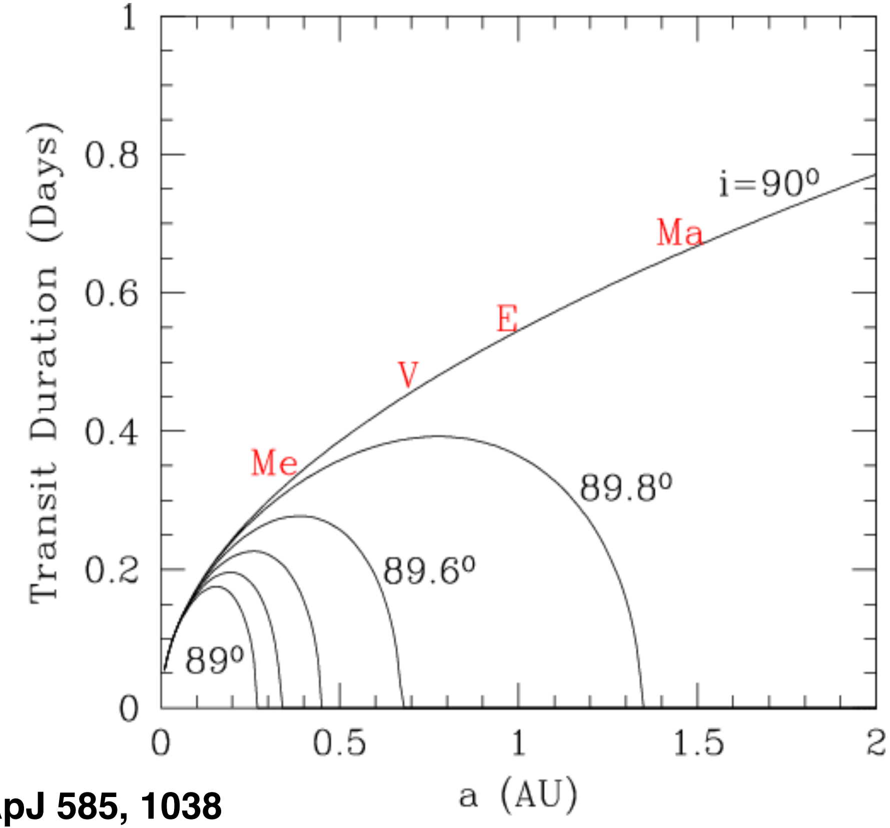
Transit Duration

$$t_T = \frac{PR_*}{\pi a}\sqrt{\left(1+\frac{R_p}{R_*}\right)^2 - \left(\frac{a}{R_*}\cos i\right)^2}\ .$$

Inclination determines impact parameter, $b = a\cos i / R_*$ hence shape of lightcurve .







Vietri Sul Mare 2015 **Seager & Mallen-Ornelas 2003, ApJ 585, 1038**

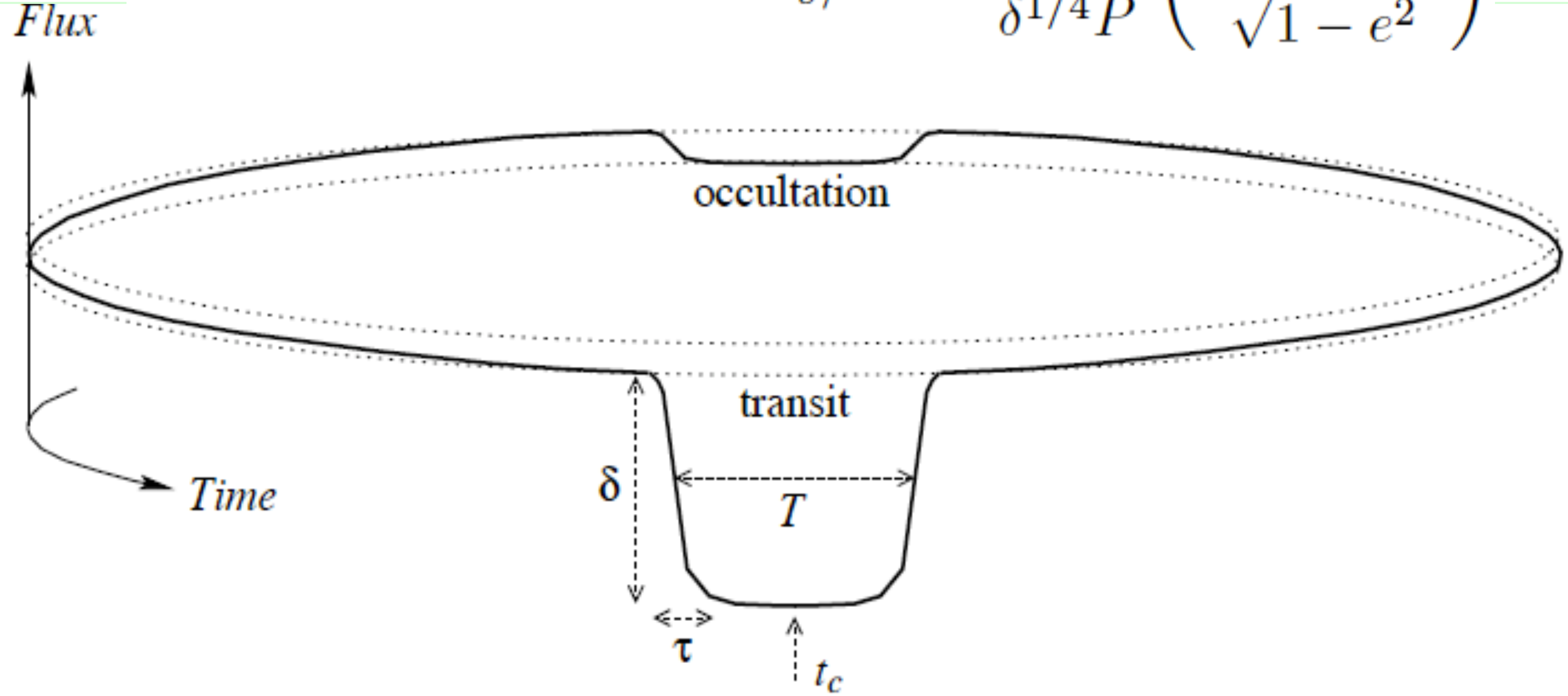
Fundamental derived quantities

- **Winn 2008, IAU Symp. 253**

$$\text{Radius ratio} \quad R_p/R_s \approx \sqrt{\delta},$$

$$\text{Impact parameter} \quad b \approx 1 - \sqrt{\delta}\frac{T}{\tau},$$

$$\text{Scaled stellar radius} \quad R_s/a \approx \frac{\pi\sqrt{T\tau}}{\delta^{1/4}P}\left(\frac{1 + e\sin\omega}{\sqrt{1-e^2}}\right)$$



Model-independent parameters

- **Winn 2008, IAU Symp. 253**

$$
\begin{aligned}
\text{Radius ratio} \quad R_p/R_s &\approx \sqrt{\delta}, \\
\text{Impact parameter} \quad b &\approx 1 - \sqrt{\delta}\frac{T}{\tau}, \\
\text{Scaled stellar radius} \quad R_s/a &\approx \frac{\pi\sqrt{T\tau}}{\delta^{1/4}P}\left(\frac{1+e\sin\omega}{\sqrt{1-e^2}}\right)
\end{aligned}
$$

$$
\text{Stellar mean density} \quad \rho_s \approx \frac{3P}{\pi^2 G}\left(\frac{\sqrt{\delta}}{T\tau}\right)^{3/2}\left[\frac{1-e^2}{(1+e\sin\omega)^2}\right]^{3/2}
$$

$$
\text{Planetary surface gravity} \quad g_p \approx \frac{2\pi K_s}{P}\frac{\sqrt{1-e^2}}{\delta(R_s/a)^2\sin i},
$$

(1) Spectral Type gives star mass and radius.

(2) Period (+ Kepler's law) gives orbit size.

(3) Depth of transit gives planet radius.

Models of planets with masses between ~ 0.1 M_J and 10 M_J, have almost the same radii (i.e. a flat mass-radius relation).

-> Giant planets transiting solar-type stars expected to have transits depths of around 1%

(4) Impact parameter $b = a \cos(i)/R_*$, determined from the shape of the transit, gives a measure of inclination angle.

(5) Bottom of light curve is not flat in all wave bands, providing a measure of stellar limb-darkening

(6) Since inclination is measured, can measure mass, not just lower limit $m_p \sin(i)$, from the radial velocity data.

Summary

- Transit probability is ~ R_*/a
  - 10% for hot Jupiter, 0.5% for Earth transiting Sun
- Transit depth is ~ $(R_p/R_*)^2$
  - 1% for Jupiter transiting Sun
- Transit duration is ~ R_*/a
  - Gives stellar density via Kepler 3
- Transit depth, duration and radial acceleration of star yield planetary surface gravity.
- To get planet density, need to know stellar radius
  - Use stellar models,
  - OR stellar angular diameter + parallax (Gaia)
- Impact parameter $b = a \cos(i)/R_*$
- Eccentricity modifies transit duration
  - Need to know orbital parameters