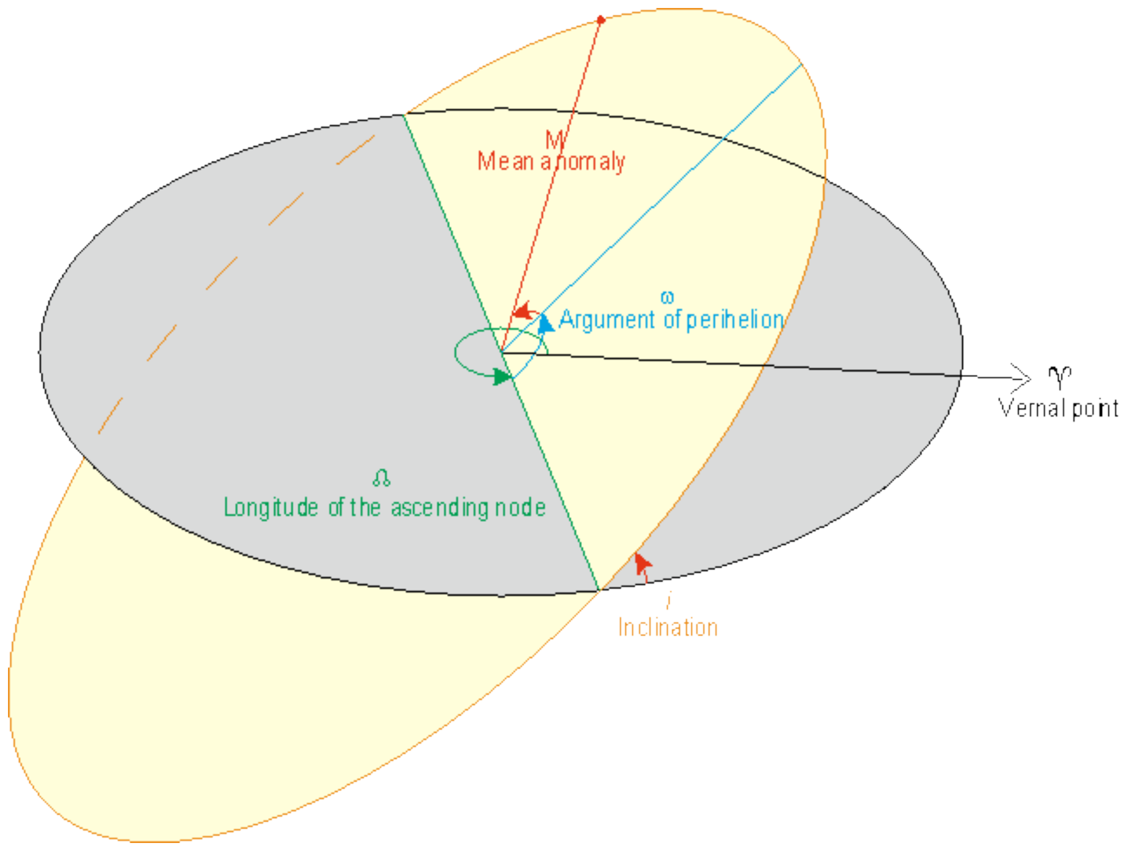
Keplerian Orbits



Not Derived by radial velocities

Ω: angle between Vernal equinox and angle of ascending node direction (orientation of orbit in sky)

i: orbital inclination (unknown and cannot be determined

Derived by radial velocities

P: period of orbit

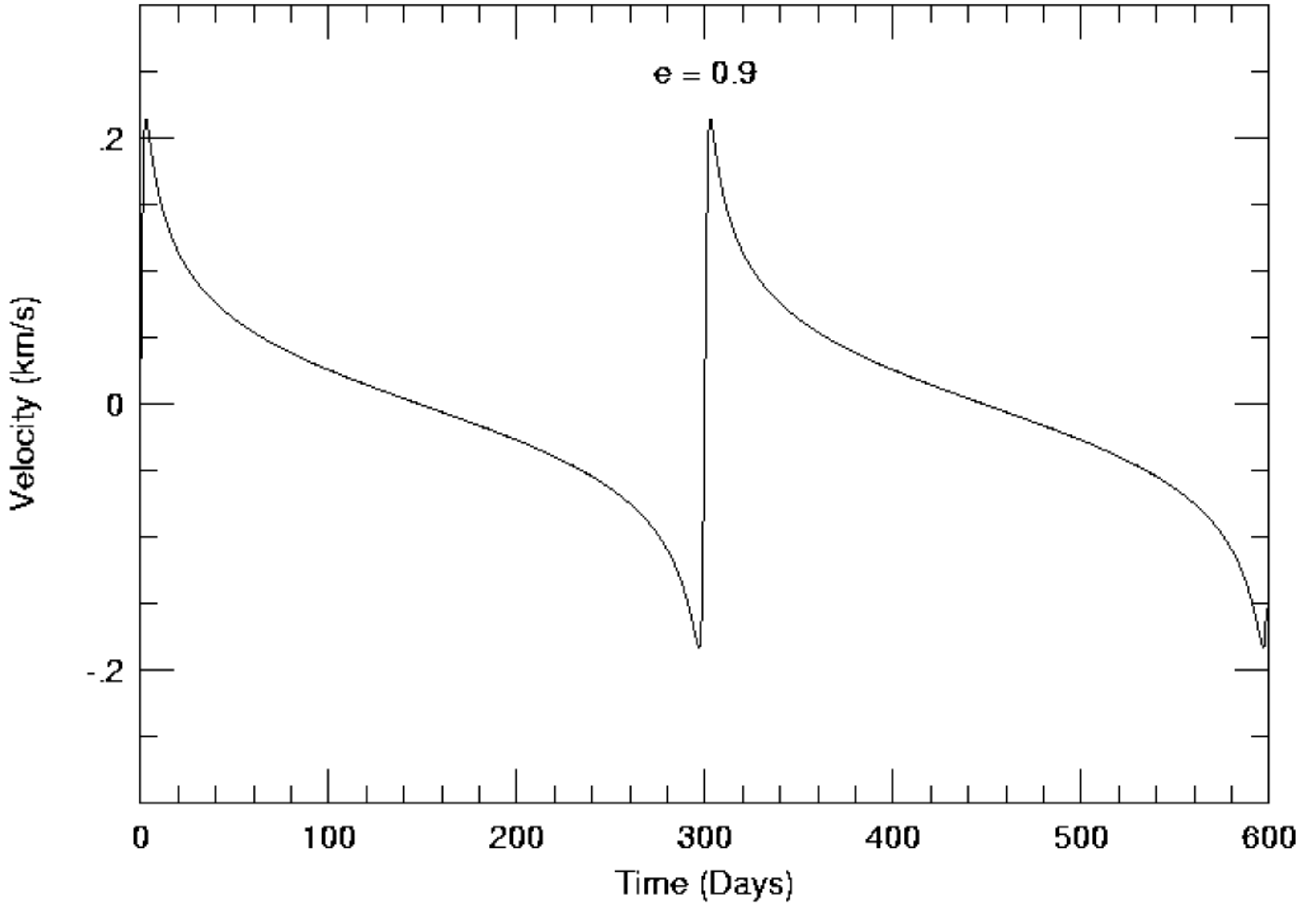
ω: orientation of periastron

e: eccentricity

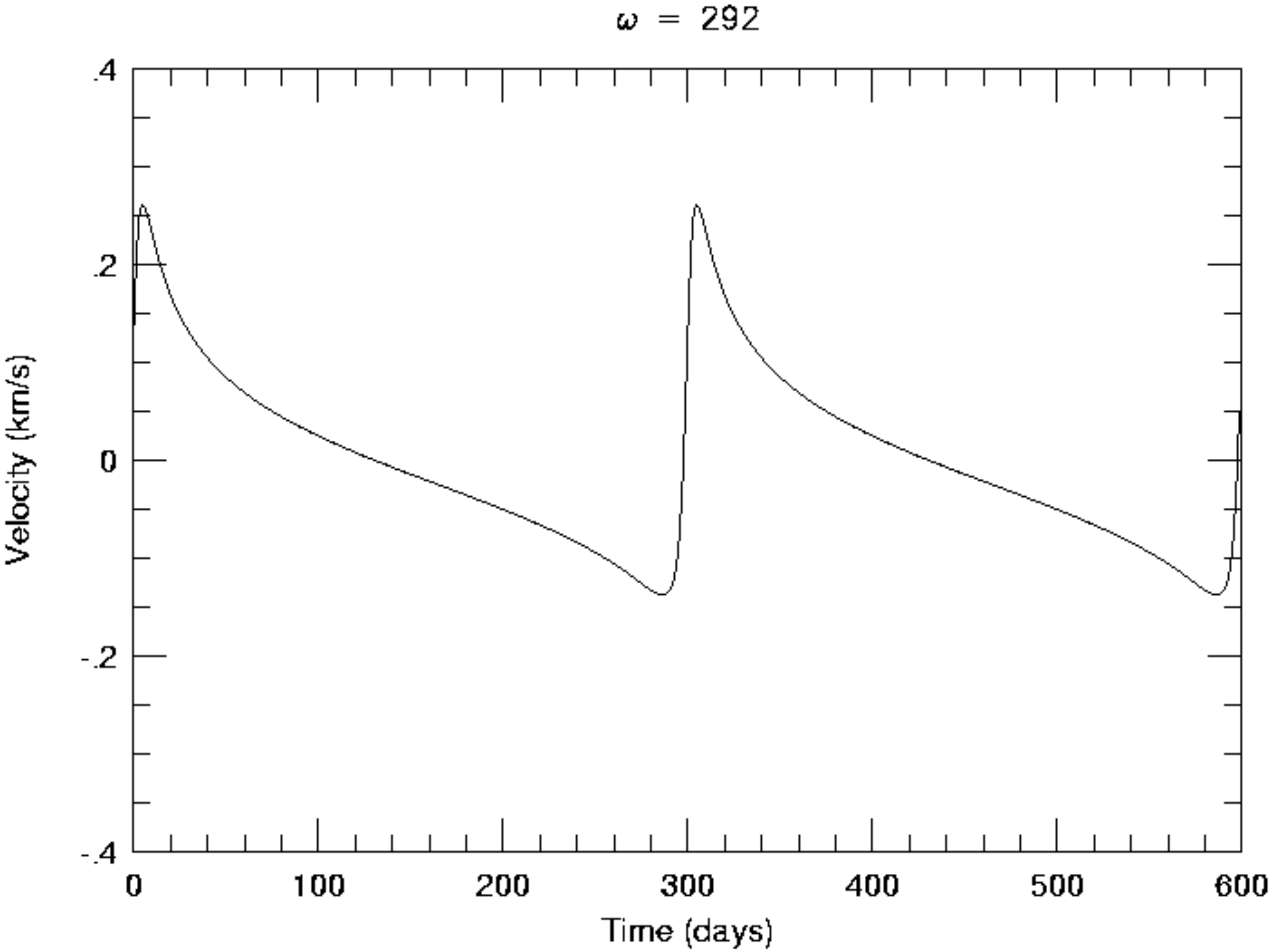
M or T: Epoch

K: velocity amplitude

Radial velocity shape as a function of eccentricity:



Radial velocity shape as a function of ω, e = 0.7 :



Resources

The Nebraska Astronomy Applet: An Online Laboratory for Astronomy

http://astro.unl.edu/naap/

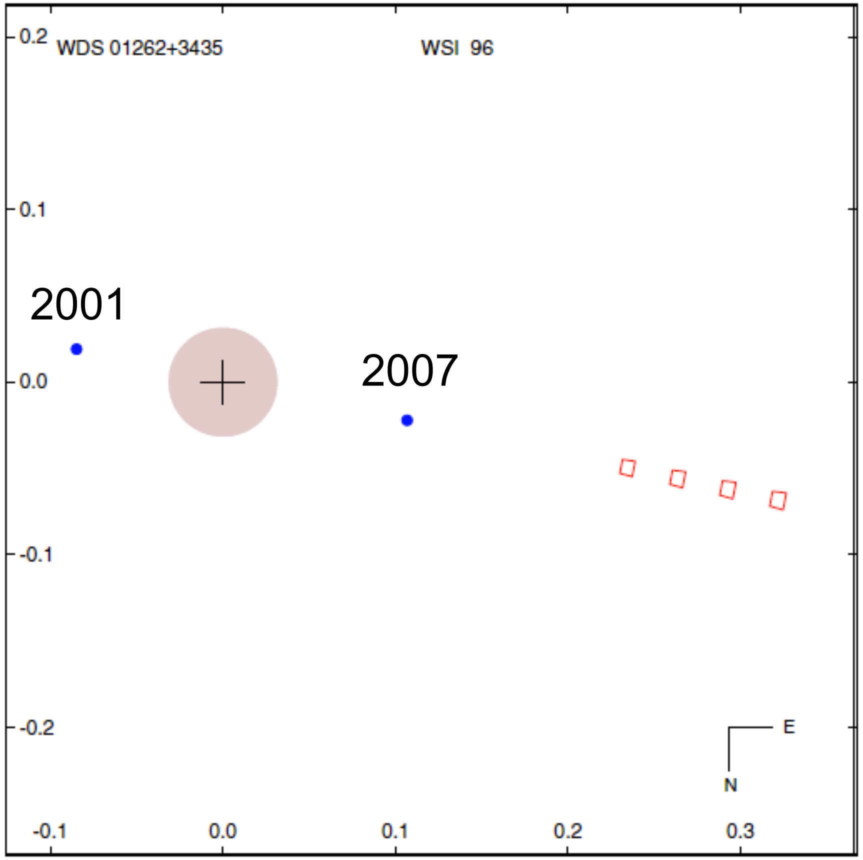
http://astro.unl.edu/animationsLinks.html

Pertinent to Exoplanets:

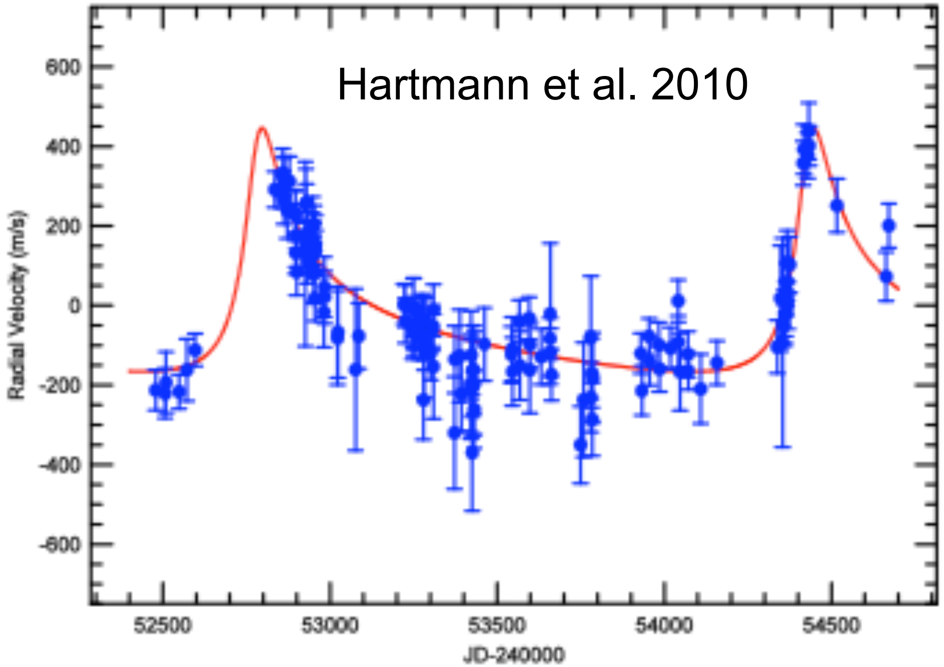
1. Influence of Planets on the Sun
2. Radial Velocity Graph
3. Transit Simulator
4. Extrasolar Planet Radial Velocity Simulator
5. Doppler Shift Simulator
6. Pulsar Period simulator

radialvelocitysimulator.htm

Mason et al. 2011



HD 8673

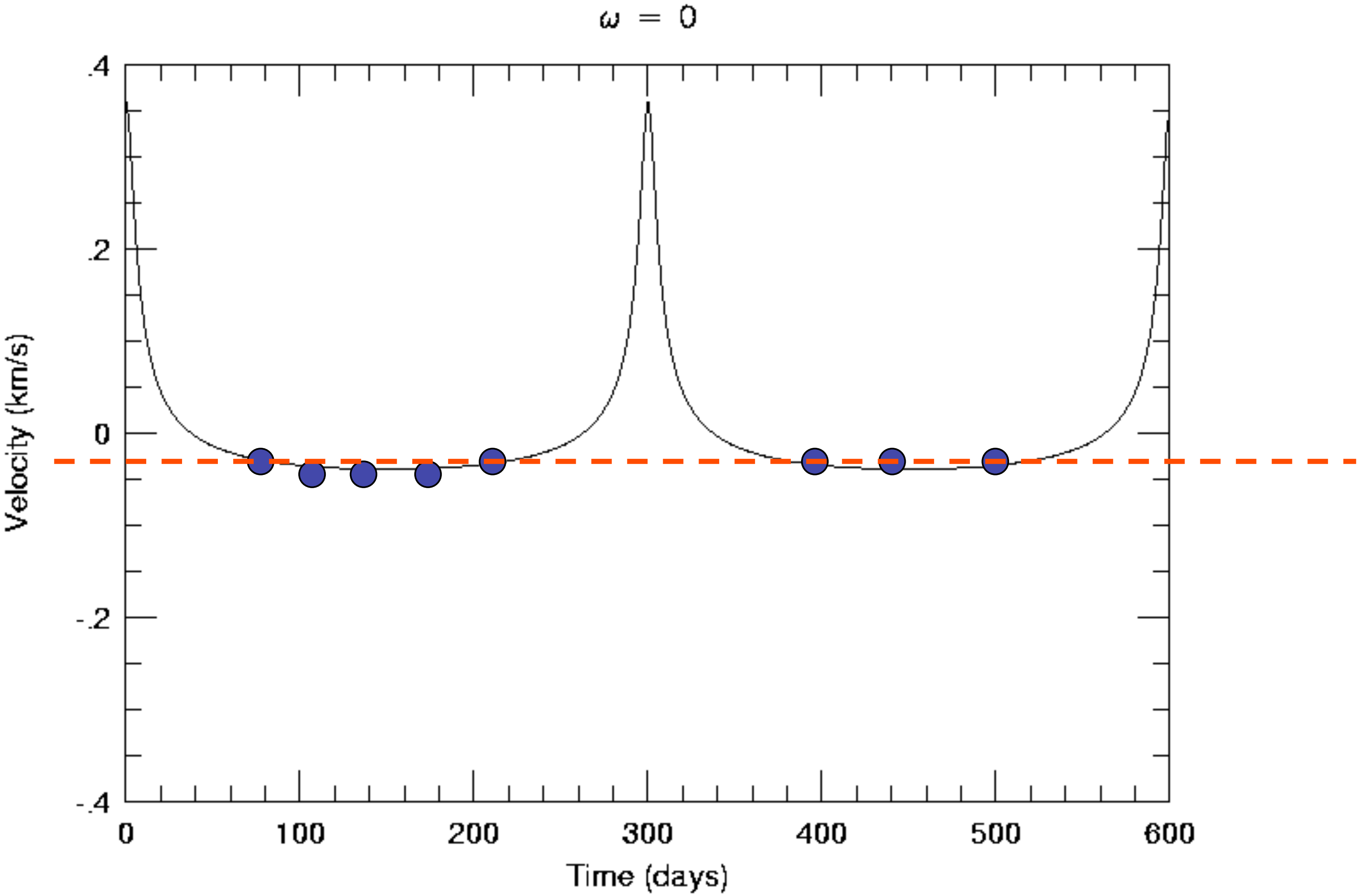


Mplanet = 14.6 $_{\mathrm{MJup}}$

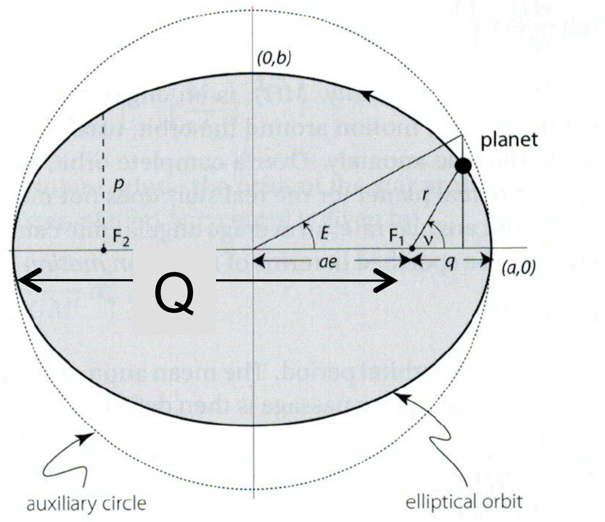
Period = 4.47 Years

ecc = 0.72

Eccentric orbit can sometimes escape detection:



With poor sampling this star would be considered constant



True anomaly: $\nu(t)$

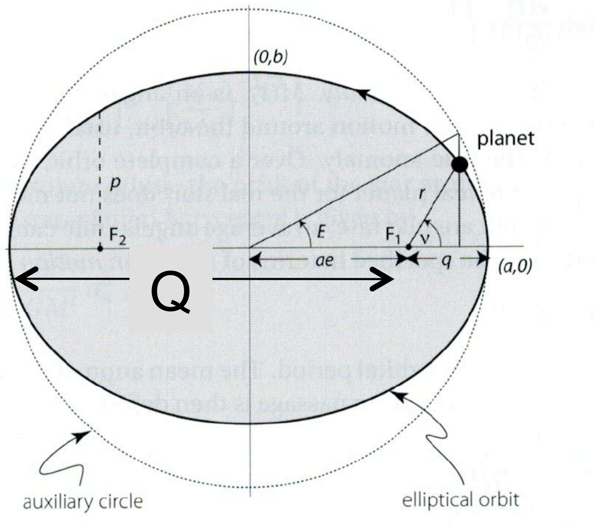
Eccentric anomaly E(t) is referred to the auxiliary circle of the ellipse

$$q = a(1-e)$$

$$Q = a(1+e)$$

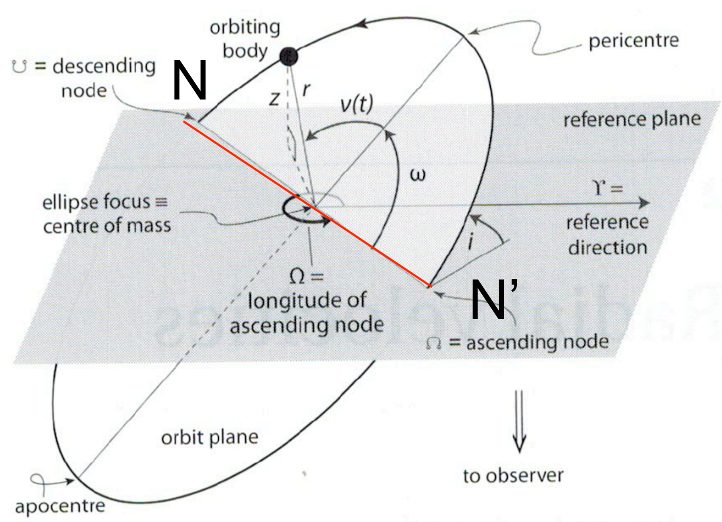
$$\cos \nu = \frac{\cos \nu(t) - e}{1 - e \cos E(t)}$$

see Exoplanets Handbook



$$r = \frac{a\,(1 - e^2)}{1 + e\cos\nu} \quad (1)$$

$$r^2 \frac{d\nu}{dt} = \frac{2\pi a^2\,(1 - e^2)^{1/2}}{P} \quad (2)$$



component of r_1 along NN' is $r\cos(\nu+\omega)$ and perpendicular is $r\sin(\nu + \omega)$

component of r_1 along line of sight is $r_1\cos(\theta_1+\omega)\sin i$

Rate of change in radial component:

$$V_o = \sin i \left[r_1 \cos(\nu + \omega) \frac{d\nu}{dt} + r_1 \sin(\nu + \omega) \frac{dr_1}{dt} \right] + \gamma$$

Use (1) and (2) to eliminate the time derivatives

$$V_o = K_1 \left[\cos(\nu + \omega) + e \cos \omega \right] + \gamma^1$$

$$K_1 = \frac{2\pi a_1 \sin i}{P(1 - e^2)^{1/2}}$$

[1]Note: we are only concerned with relative radial velocities, so γ is often just the instrumental offset.

Let A_1 be the absolute value of the maximum velocity and B_1 the absolute value of the “maximum” negative velocity

$$A_1 = K_1(1 + e \cos \omega)$$

$$B_1 = K_1(1 - e \cos \omega)$$

$$K_1 = ½\, K_1(A_1 + B_1)$$

The Mass Function

$$\frac{G}{4\pi^2}\,(M_1 + M_2)\,P^2 = (a_1 + a_2)^3$$

$$= a_1^{\,3}\,(1 + \frac{a_2}{a_1})^3$$

$$= a_1^{\,3}\,(1 + \frac{M_1}{M_2})^3$$

$$\frac{a_2}{a_1} = \frac{M_1}{M_2}$$

$$\frac{G}{4\pi^2}\,(M_1 + M_2)\,P^2 \sin^3 i = a_1^{\,3}\,\sin^3 i\,\left(\frac{M_1 + M_2}{M_2}\right)^3$$

$$\frac{GP^2}{4\pi^2}\left(\frac{M_2^3\,\sin^3 i}{M_1 + M_2}\right)^2 = a_1^{\,3}\,\sin^3 i$$

The Mass Function

$$a_1 \sin i = \frac{K_1 \, P(1 - e^2)^{1/2}}{2\pi}$$

$$\frac{GP^2}{4\pi^2} \frac{M_2^3 \sin^3 i}{\left(M_1 + M_2\right)^2} = \frac{K_1^3 \, P^3(1 - e^2)^{3/2}}{(2\pi)^3}$$

$$\frac{M_2^3 \sin^3 i}{\left(M_1 + M_2\right)^2} = \frac{K_1^3 \, P(1 - e^2)^{3/2}}{(2\pi G)}$$

The Mass Function

$$f(m) = \frac{(m_p \sin i)^3}{(m_p + m_s)^2}$$

$$\frac{M_2^3 \sin^3 i}{(M_1 + M_2)^2} = \frac{K_1^3 P(1 - e^2)^{3/2}}{(2\pi G)}$$

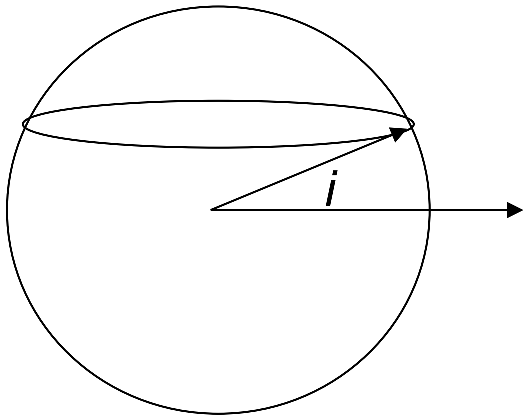
m_p = mass of planet

m_s = mass of star

The Orbital Inclination

We only measure m sin i, a lower limit to the mass.

What is the average inclination?



$$P(i)\,di = 2\pi \sin i \; di$$

The probability that a given axial orientation is proportional to the fraction of a celestrial sphere the axis can point to while maintaining the same inclination

The Orbital Inclination

$$P(i)\,di = 2\pi \sin i\, di$$

Mean inclination:

$$\langle \sin i \rangle = \frac{\int_0^{\pi} P(i) \sin i \, di}{\int_0^{\pi} P(i)\, di} = \pi/4 = 0.79$$

Mean inclination is 52 degrees and you measure 80% of the true mass

$$P(i)\,di = 2\pi \sin i \, di$$

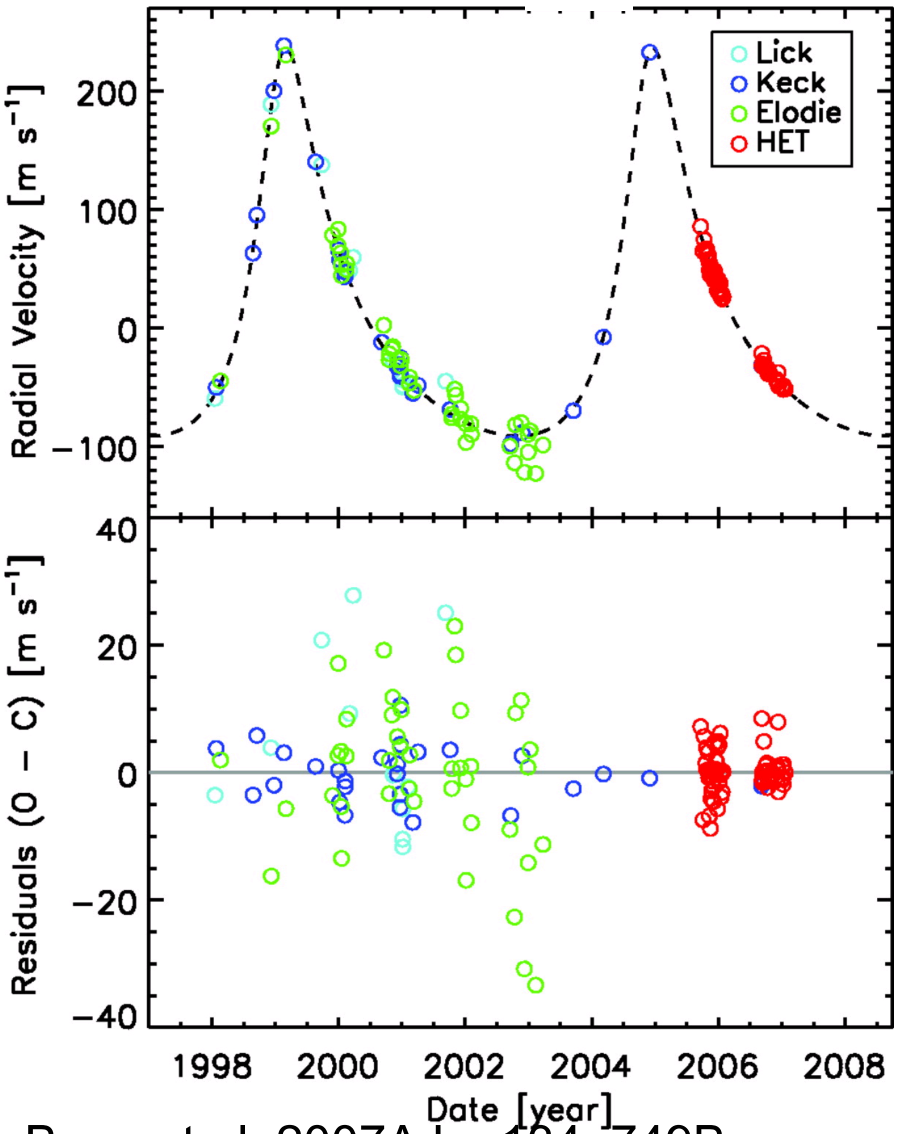
Probability $i < \theta$:

$$P(I < \theta) = \frac{2\int_0^{\theta} P(i)\,di}{\int_0^{\pi} P(i)\,di} = (1 - \cos\theta)$$

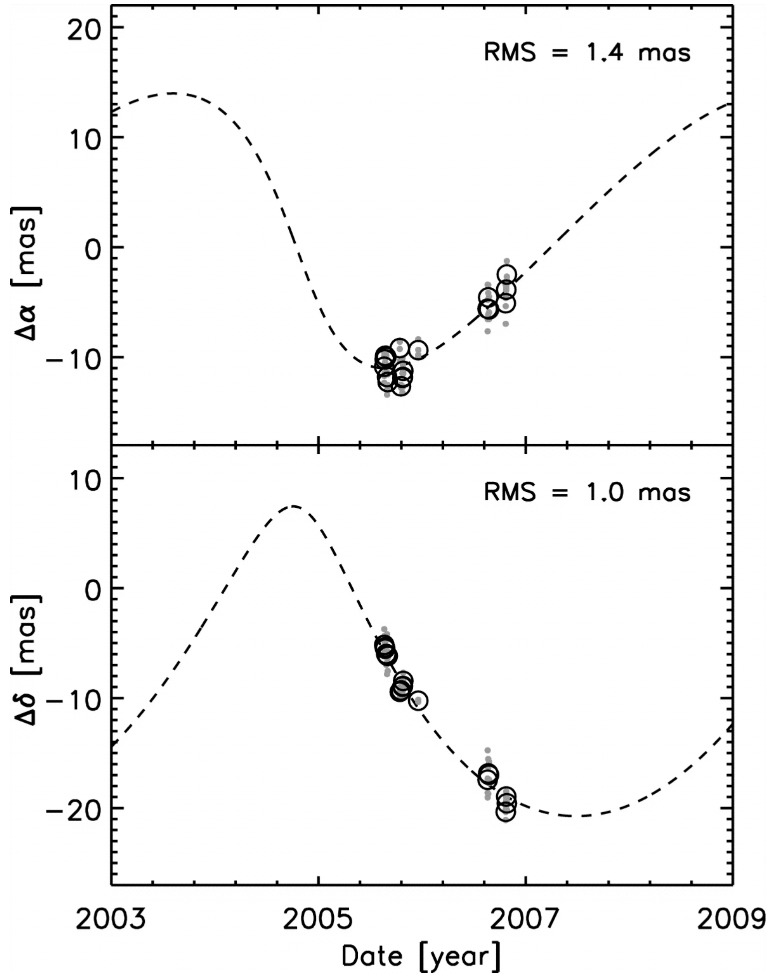
$\theta < 10$ deg : $P = 0.03$ $\quad$ ($\sin i = 0.17$)

One of our planets is missing: sometimes you need the true mass!

HD 33636 B



Bean et al. 2007AJ....134..749B



P = 2173 d

Msini = 10.2 M_{Jup}

i = 4 deg → m = 142 M_{Jup}

= 0.142 M_{sun}

$$P(i)\,di = 2\pi \sin i \, di$$

<u>But for the mass function $\sin^3 i$ is what is important :</u>

$$\langle \sin^3 i \rangle = \frac{\int_0^{\pi} P(i) \sin^3 i \, di}{\int_0^{\pi} P(i)\, di} = 0.5 \int_0^{\pi} \sin^4 i \, di$$

$$= 3\pi/16 = 0.59$$

GaussFit

A System for Least Squares and Robust Estimation

http://clyde.as.utexas.edu/Gaussfit.html

Orbit fitting with Gaussfit: environment file (env)

```
results =     'RVRES'
data1   =     'hd137510.in’          <------ data file
param1  =     'gammas.prm'           <------ γ (offsets) file
param2  =     'astr_rvA.prm'         <------ orbit parameters file
param3  =     'rv.prm'               <------ RV amplitude (K) file
tol     =     0.0001
iters   =     20.0
prmat   =     0.0
prvar   =     1.0
sigma   =     1.9438622700469972
scale   =     1.9438622700469974
END
```

Orbit fitting with Gaussfit: orbit parameters (astr_rvA.prm)

P1 T1 ecc1 wrv1 delta_ecc1 delta_wrv1 delta_T1 delta_P1 sigma_ecc1 sigma_wrv1 sigma_T1 sigma_P1

double double double double double double double double double double double double

799.06140275208565 51781.351677079524 0.3786344601133155
31.899848129546125 0.0006640266885857 -0.0810250490409570
-1.3402733107337730 1.1614099687610568 0.0269623416328822
4.6569162312834642 50.052739351717072 41.260892358499738

P1 = period in days

T1 = Epoch

ecc1 = eccentricity

wrv1 = omega

Things to note

- For circular orbits omega is superfluous as it can be absorbed by T0 (i.e. T1).
- For transiting planets where you use the epoch of transit for T_0 (i.e. T1), omega (wrv1) = 90 degrees. This is true even for circular orbits. This ensures that the RV is in the proper phase with the transit ephemeris

Orbit fitting with Gaussfit: rv amplitude (rv.prm)

K1 sigma_K1 delta_K1
double double double
0.500731315026525 0.0219221878492379
-0.0000534493161382

Orbit fitting with Gaussfit: gamma velocity (gamma.prm)

gamma delta_gamma sigma_gamma
double double double
0.0549563711190492 -0.0005342159945130
0.0231322727323946

Orbit fitting with Gaussfit: model

parameter P1;   /* period in years */
constant ecc1;   /* eccentricity */
parameter T1;   /* time of pericenter passage */
constant wrv1;   /* argument of pericenter (the angle from the ascending node
to the body in the orbital plane) for RV*/
parameter K1;   /* semi-amplitude of the radial velocity curve in km/sec-1   */
parameter gamma;   /* gamma is V0 radial velocity of the center of mass constant the system */

;

```
parameter K1;   /* semi-amplitude of the radial velocity curve in km/sec-1
*/
parameter gamma1;      /* gamma is V0 radial velocity of the center of
mass
                       constant the system */
parameter gamma2;      /* gamma is V0 radial velocity of the center of
mass
                       constant the system */
parameter gamma3;      /* gamma is V0 radial velocity of the center of
mass
                       constant the system */
parameter gamma4;
parameter gamma5;
parameter gamma6;
parameter gamma7;
parameter gamma8;
parameter gamma9;
parameter gamma10;
parameter gamma11;
parameter gamma12;
parameter gamma13;
```

Orbit fitting with Gaussfit: model

```
 E = kepler(ecc1,mu*(ct-T1));      /*Solve Kepler's Equation
*/
             agamma = gamma;
             sinE = sin(E);
             cosE = cos(E);
             ecos = 1 - ecc1 * cosE;
             cosv = (cosE - ecc1)/ecos;
             sinv = param*sinE/ecos;
             cosvw = cosv * coswrv - sinv * sinwrv;
             vorb = agamma + K1*(ecc1*coswrv+cosvw);
```

Gaussfit environment files with multiple data sets

```
results =      'RVRES'
data1   =      'keck1.in'
data2   =      'keck2.in'
data3   =      'keck3.in'
data4   =      'keck4.in'
data5   =      'keck5.in'
data6   =      'keck6.in'
data7   =      'keck7.in'
param2  =      'gammas.prm'
param3  =      'astr_rvA.prm'
param4  =      'rv.prm'
tol     =      0.0001
iters   =      20
prmat   =      0
prvar   =      1
sigma   =      1.692582597927886
scale   =      1.692582597927886
END
```

gamma1 gamma2 gamma3 gamma4 gamma5 gamma6 gamma7
gamma8 gamma9 gamma10 gamma11 gamma12 gamma13 gamma14
gamma15 gamma16 gamma17 gamma18 gamma19 gamma20 gamma21
gamma22 gamma23 gamma24 gamma25 sigma_gamma12
sigma_gamma10 sigma_gamma17 sigma_gamma21 sigma_gamma9
sigma_gamma6 sigma_gamma13 sigma_gamma20 sigma_gamma19
sigma_gamma3 sigma_gamma8 sigma_gamma4 sigma_gamma16
sigma_gamma18 sigma_gamma5 sigma_gamma7 sigma_gamma11
sigma_gamma22 sigma_gamma15 sigma_gamma2 sigma_gamma14
sigma_gamma1 sigma_gamma23 sigma_gamma24 sigma_gamma25

double double double double double double double double double
double double double double double double double double double
double double double double double double double double double
double double double double double double double double double
double double double double double double double double double
double double double double double

0 0 0 0 0 0 0

Up to 27 gammas allowed

Gaussfit data file (orbit.in, data.in)

```
file    item    jd_rv                  RV                RVerr   VA      VA_VA   _VA
double  double  double  double  double  double  double  double
 1       1       52039.6016      107.20284271    15      0.107203        0.000225
0.009932542049228875
 1       2       52040.5938      118.53701019    15      0.118537        0.000225
0.01369003057627049
 1       3       52040.5977      89.3843689      15      0.0893844       0.000225
-0.01550237677517226
 1       4       52041.582       115.35044098    15      0.11535         0.000225
-0.002575948254958333
```

RV: radial velocity in m/s

RVerr : radial velocity error in m/s

VA : radial velocity in km/s

VA_VA: σ^2 in km/s

awk script "gfprep" puts a data file in m/s into proper format

Steps for orbit fitting with Gaussfit:

1. Determine the period (Scargle, Period04)
2. Get an estimate of the amplitude (eye, sin fitting, Period04)
3. Guess T0
4. Set e = 0, omega= 0
5. Vary parameters individually. i.e. first T0, then P, then K. Typically vary a parameter say, P, with K and gamma
6. Once you find best solution, then vary eccentricity. Sometimes you get negative eccentricities, if you do then start with some non-zero eccentricity
7. Vary sequentially omega, ecc, P, T1
8. When you are close vary everything!

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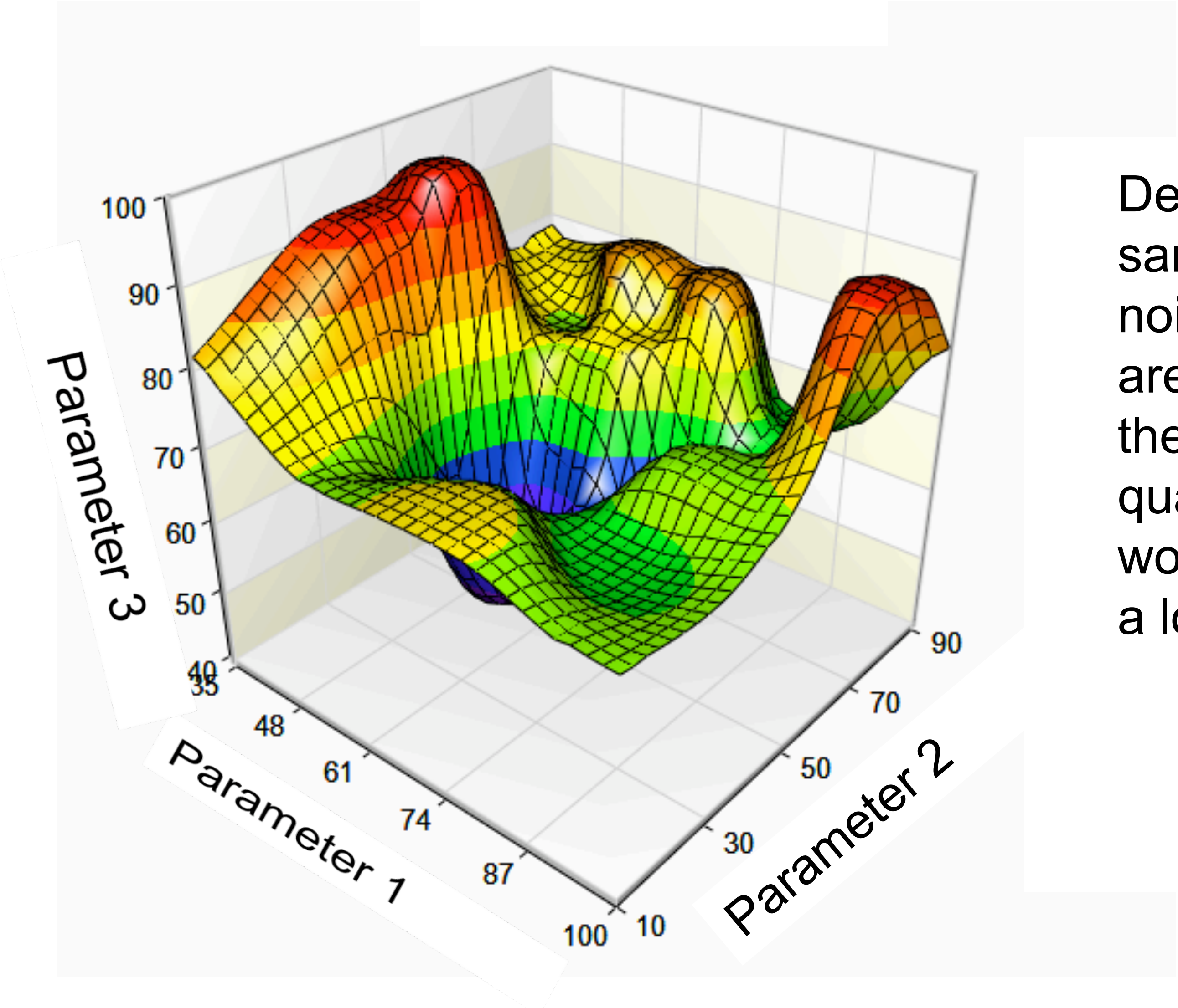
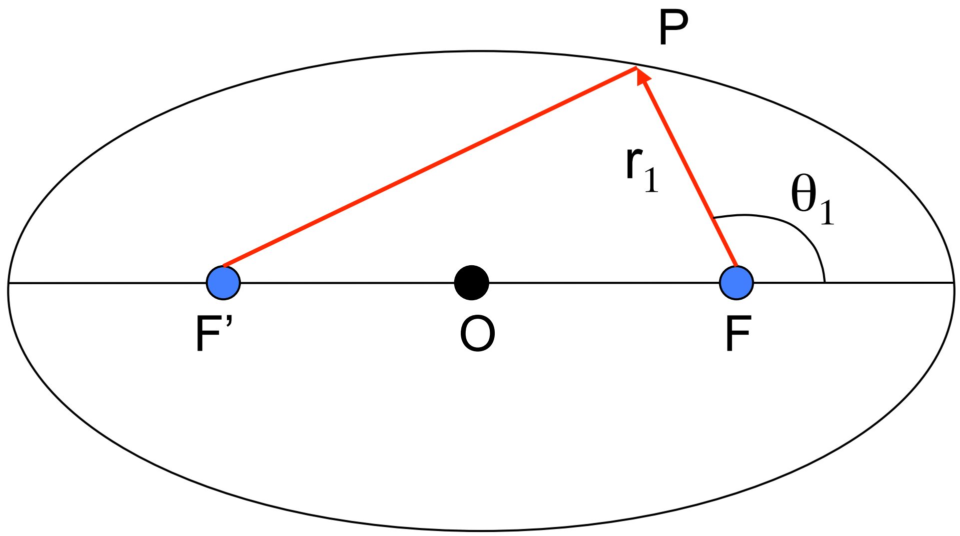
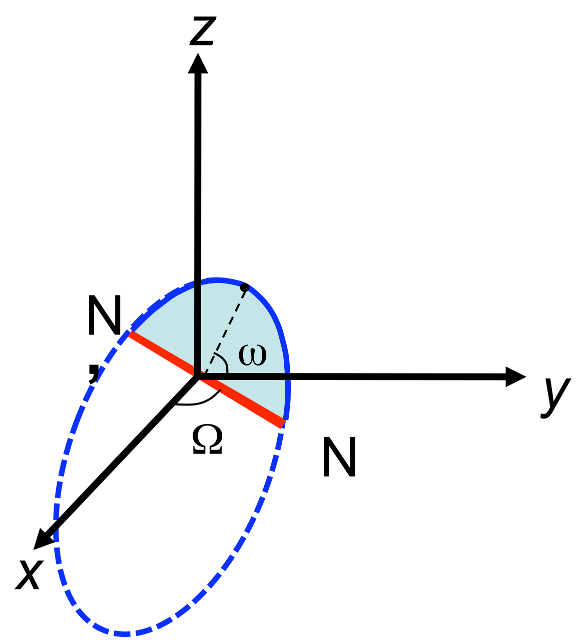


Depending on the sampling and the level of noise, some parameters are better determined then others. For poorer quality data you should worry whether you are in a local or global minimum



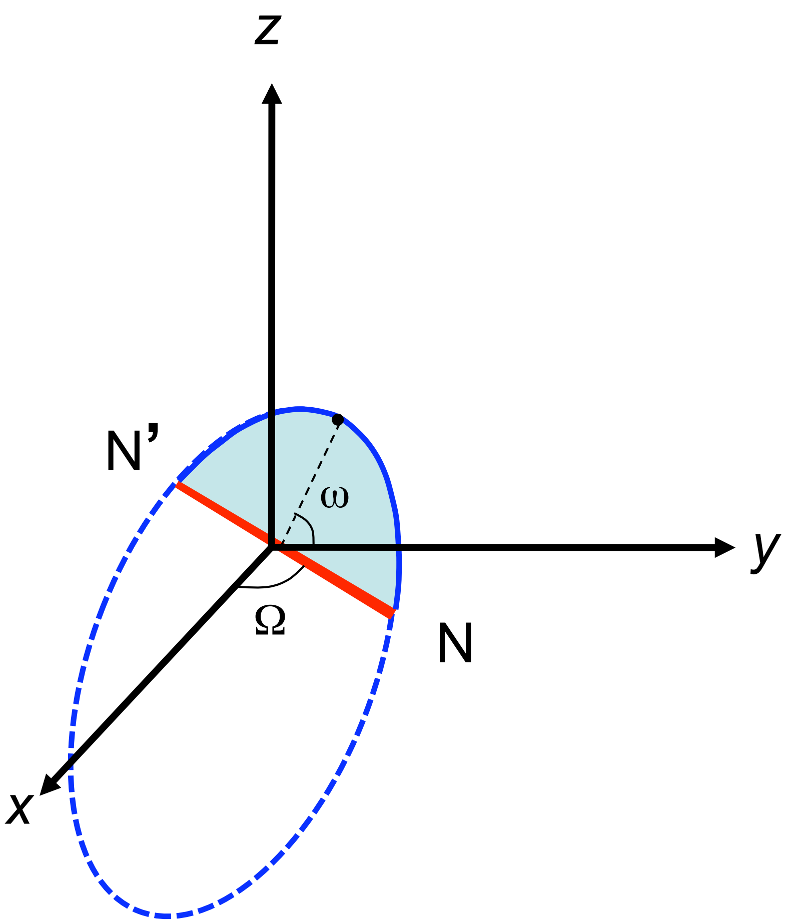


component of r_1 along NN' is $r_1\cos(\theta_1+\omega)$ and perpendicular is $r_1\sin(\theta_1 + \omega)$

component of r_1 along line of sight is $r_1\cos(\theta_1+\omega)\sin i$

$$r = \frac{a\,(1 - e^2)}{1 + e\cos\theta} \quad (1)$$

$$r^2\,\frac{d\theta_1}{dt} = \frac{2\pi a^2\,(1 - e^2)^{½}}{P} \quad (2)$$



x-y is the plane of the sky

z is towards observer

NN' is the line of nodes where the orbital plane intercepts the x-y plane

dashed line is part of orbit below the x-y plane

Ω is the position angle of node

ω is the longitude of periastron

radialvelocitysimulator.htm