Max Planck Institute for Solid State Research

Novel Non-Equilibrium Dynamics in Superconductors: Induced Superconductivity and Higgs Modes

Nikolaj Bittner

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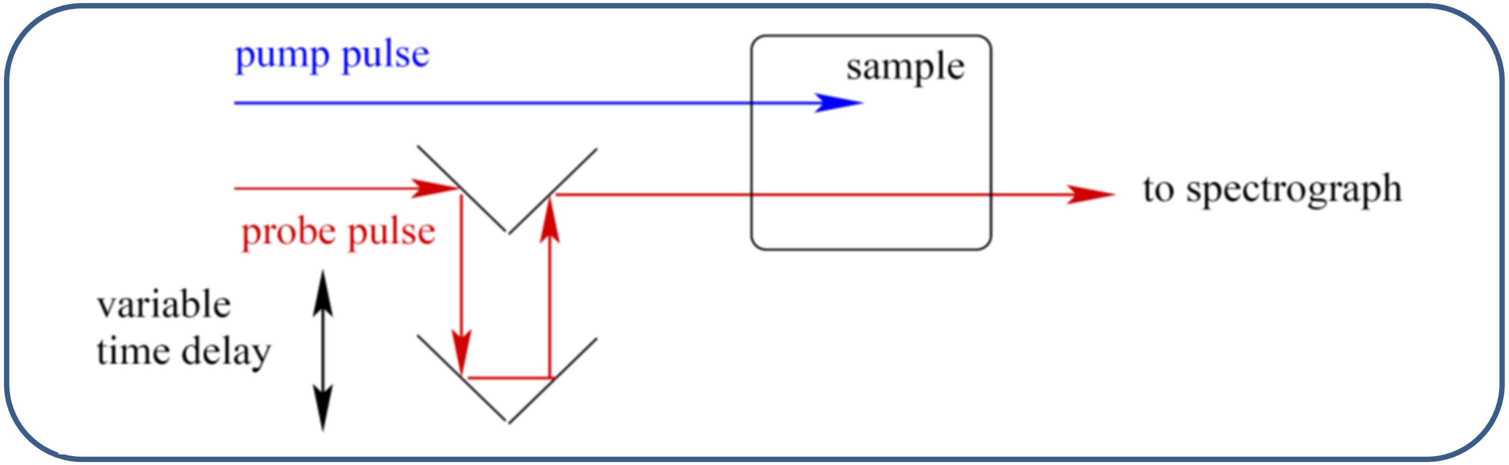
Takami Tohyama (Tokyo US), and Dirk Manske (MPI)
Exp.: Stefan Kaiser (MPI)

Introduction: pump-probe technique



» Pump-probe spectroscopy

⇨ **Pump pulse:** excitation of the system with an **intence pulse**. **Induces dynamics.**

⇨ **Probe pulse:** measuring with **less intence pulse** after time delay Δt.

Makes „snapshots" of a nonequilibrium state

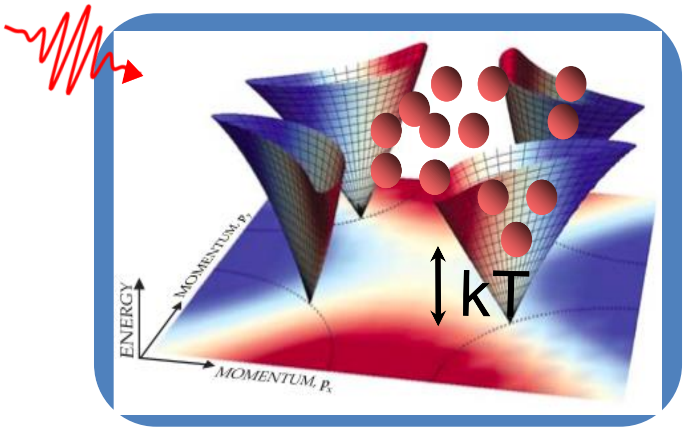
» Two kinds of information can be extracted:

⇨ **Time domain:** conductivity change $\Delta\sigma$, depending on time delay Δt

⇨ **Energy domain:** change in the conductivity spectra

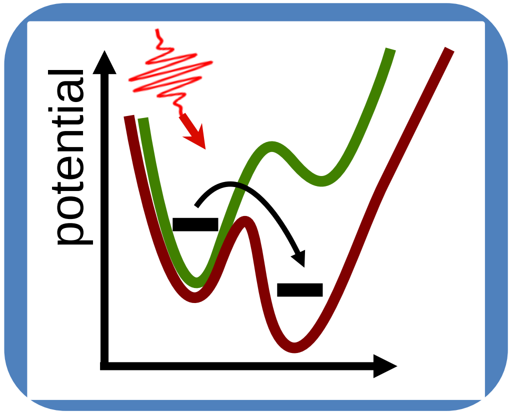
Types of nonequilibrium experiments

nonequilibrium spectroscopy



- Recovery of SC condensate
- Cooper pair formation dynamics
- SC glue

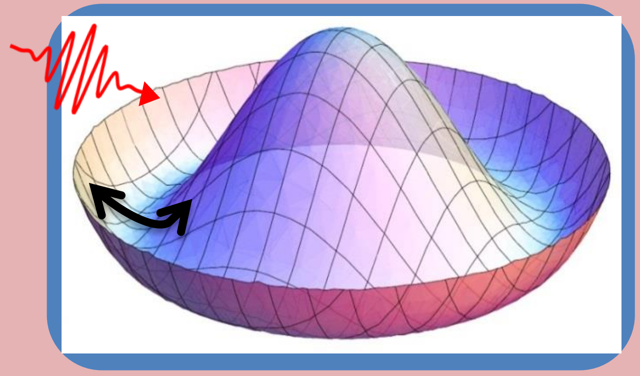
optical control



- New transient ground state

Outline

New! Higgs spectroscopy



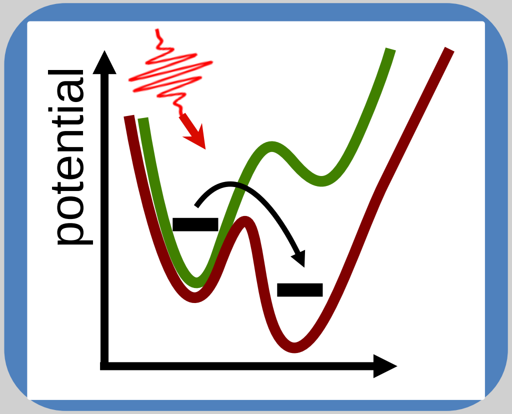
» Direct detection of the SC gap

Part 2

Order parameter oscillations in Superconductors

H. Krull, N. Bittner, G. S. Uhrig, D. Manske, and A.P. Schnyder, Nat. Commun. 7:11921 (2016)

optical control



» New transient ground state

Part 1

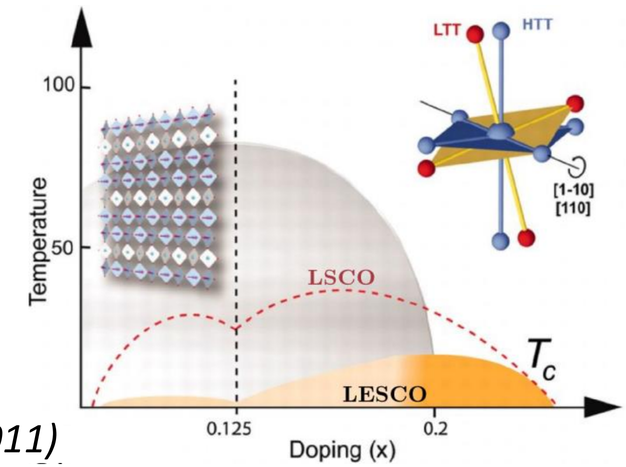
Induced Superconductivity in the extended Hubbard Model

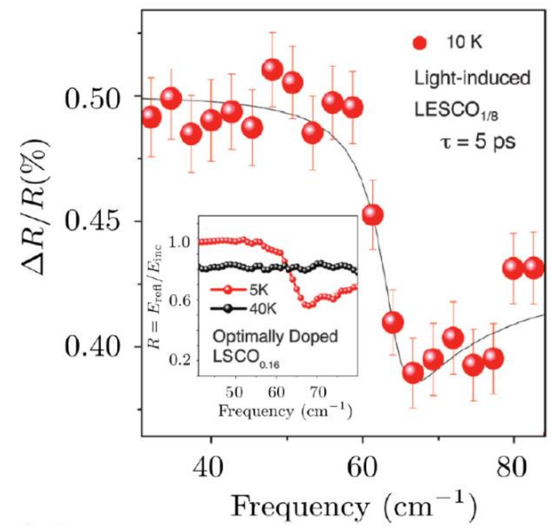
N. Bittner, T. Tohyama, and D. Manske, in preparation

Motivation: Induced superconductivity?

I. Stripe-Ordered Cuprate $LESCO_{1/8}$

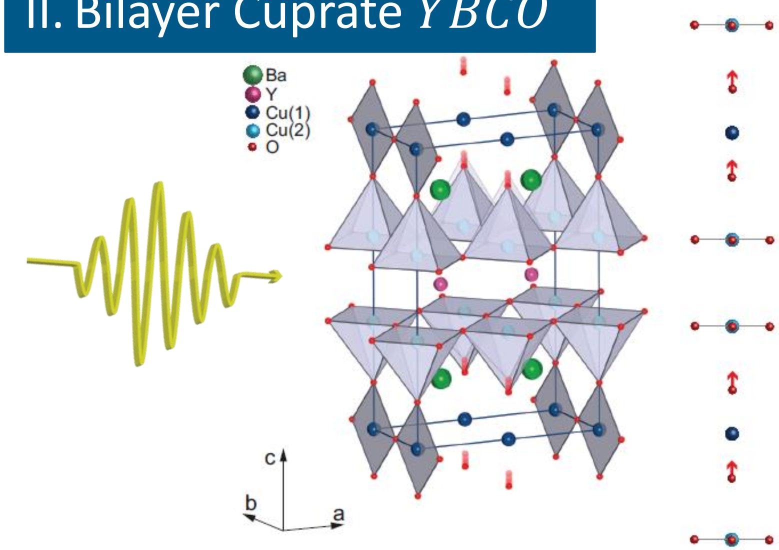
Melting of the competing order

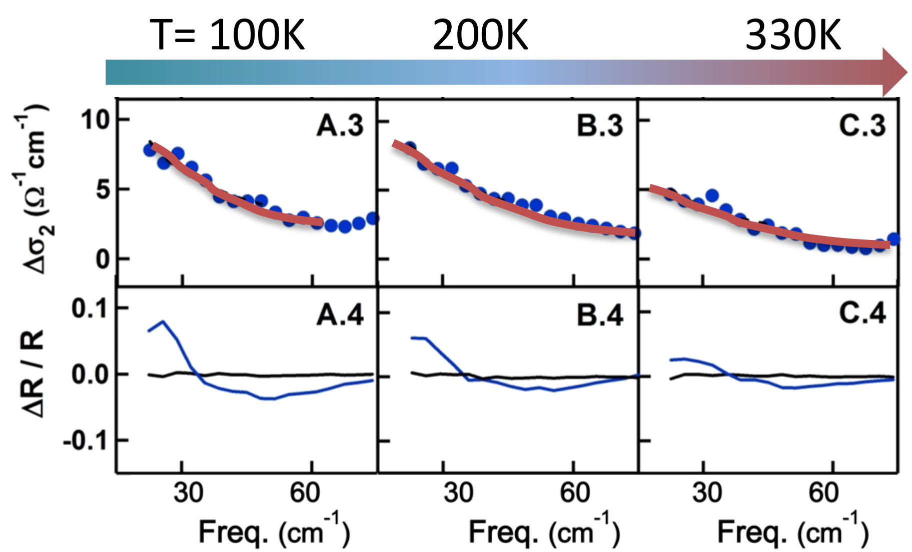




D. Fausti et al. Science ***331****, 189 (2011)*

II. Bilayer Cuprate $YBCO$





A. Cavalleri et al. Nature, ***516****, 71 ; S. Kaiser et al. PR B,* ***89****, 184516 (2014)*

Motivation: Existing theoretical approaches

» Suppression of the **competing order**

Z. M. Raines et al., PR B ***91****, 184506 (2015)*

A. A. Patel and A. Eberlein, PR B ***93****, 195139 (2016)*

» **Amplification of the superconducting correlations** (e.g. due to the increase of the DoS near the Fermi level)

M. A. Sentef et al., PR B ***93****, 144506 (2016)*

» **Nonlinear phononics**. Driving of Raman modes leads to the cubic coupling to an infrared phonon mode, which causes induced superconductivity

R. Mankowski et al., Nature ***516****, 71 (2014)*

→ Induced Superconductivity after pumping?

→ Is the underlying physics special for cuprates?

→ Transient Meissner effect? Can we calculate this?

1D Hubbard model in equilibrium

Our strategy:

» calculate correlation function

» calculate response function

1D Hubbard model in a nutshell

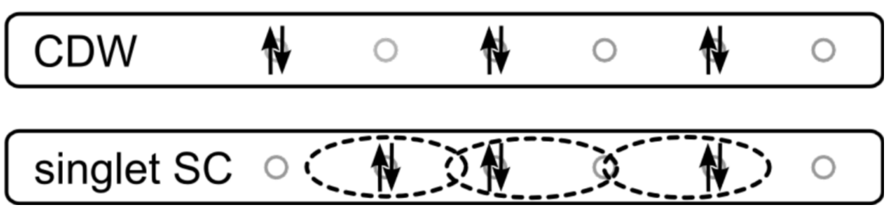
Extended Hamilton operator

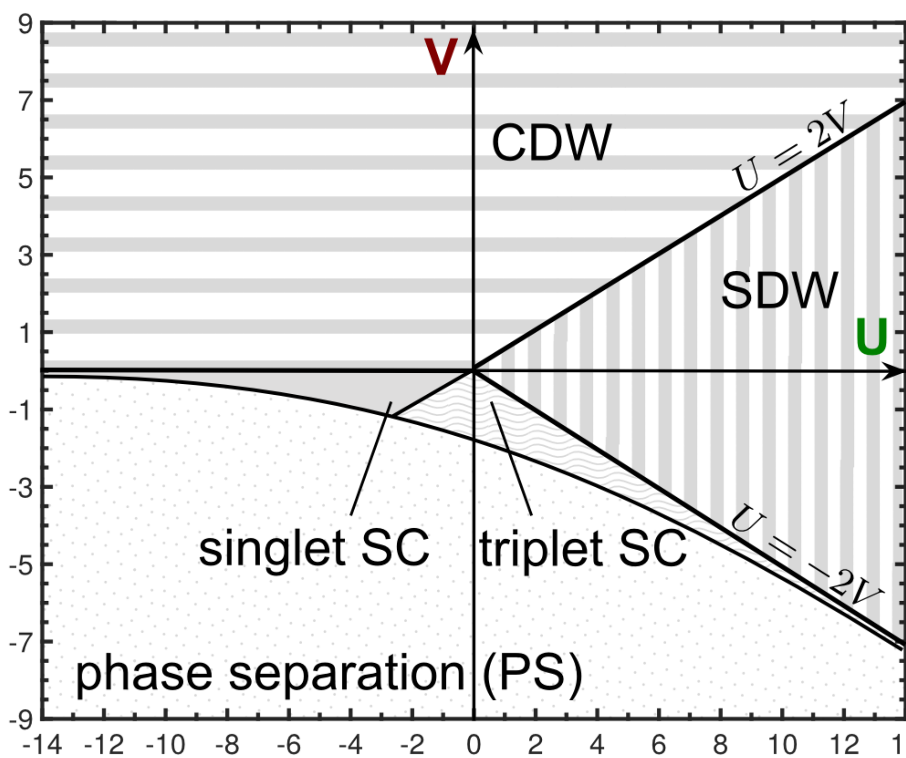
$$\hat{H}_0 = \boxed{-t_h \sum_{i,\sigma} \left(\hat{c}^\dagger_{i,\sigma}\hat{c}_{i+1,\sigma} + H.c.\right)} + \boxed{U \sum_{i} \hat{n}_{i\uparrow}\hat{n}_{i\downarrow}} + \boxed{V \sum_{\langle i,j\rangle} \hat{n}_i\hat{n}_j}$$

Kinetic term　　On-site interaction　　Nearest neighbor interaction

Phase diagram

- A "toy" model
- Well studied in equilibrium
- Very rich phase diagram





J.Solyom, Adv. Phys. ***28****, 201 (1979)*

J.Voit, PR B ***45****, 4027 (1992), Rep.Prog.Phys.* ***58****, 977 (1995)*

Two types of quenching

Extended Hamilton operator

$$\hat{H}_0 = \boxed{-t_h \sum_{i,\sigma} \left(\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i+1,\sigma} + H.c.\right)} + \boxed{U \sum_{i} \hat{n}_{i\uparrow}\hat{n}_{i\downarrow}} + \boxed{V \sum_{\langle i,j \rangle} \hat{n}_i \hat{n}_j}$$

Kinetic term | On-site interaction | Nearest neighbor interaction

Time dependence

I) Interaction quench

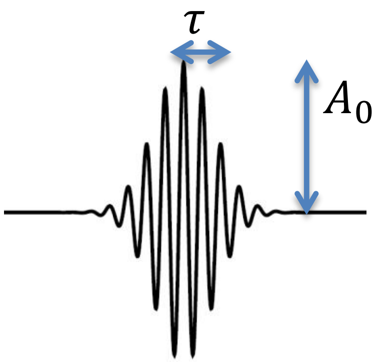
$$\boldsymbol{U} \rightarrow \boldsymbol{U} + \boldsymbol{\Delta \mathrm{U}} \cdot \Theta(t - t_0)$$

$$\boldsymbol{V} \rightarrow \boldsymbol{V} + \Delta V \cdot \Theta(t - t_0)$$

II) Pulse quench

$$A(t) = A_0 e^{-(t-t_0)^2/2\tau^2} \cos[\omega_0(t - t_0)]$$

Peierls substitution $\quad \boldsymbol{t_h} \rightarrow \boldsymbol{t_h} \cdot e^{iA(t)}$



Time dependent exact diagonalization

→ Lanczos method in time intervals δt

→ Stepwise approximation of $|\psi(t)\rangle$

$$|\psi(t + \delta t)\rangle \simeq e^{-i\hat{H}(t)\delta t}|\psi(t)\rangle \simeq \sum_{l=1}^{M} e^{-i\epsilon_l \delta t}|\phi_l\rangle\langle\phi_l|\psi(t)\rangle$$

Correlation functions

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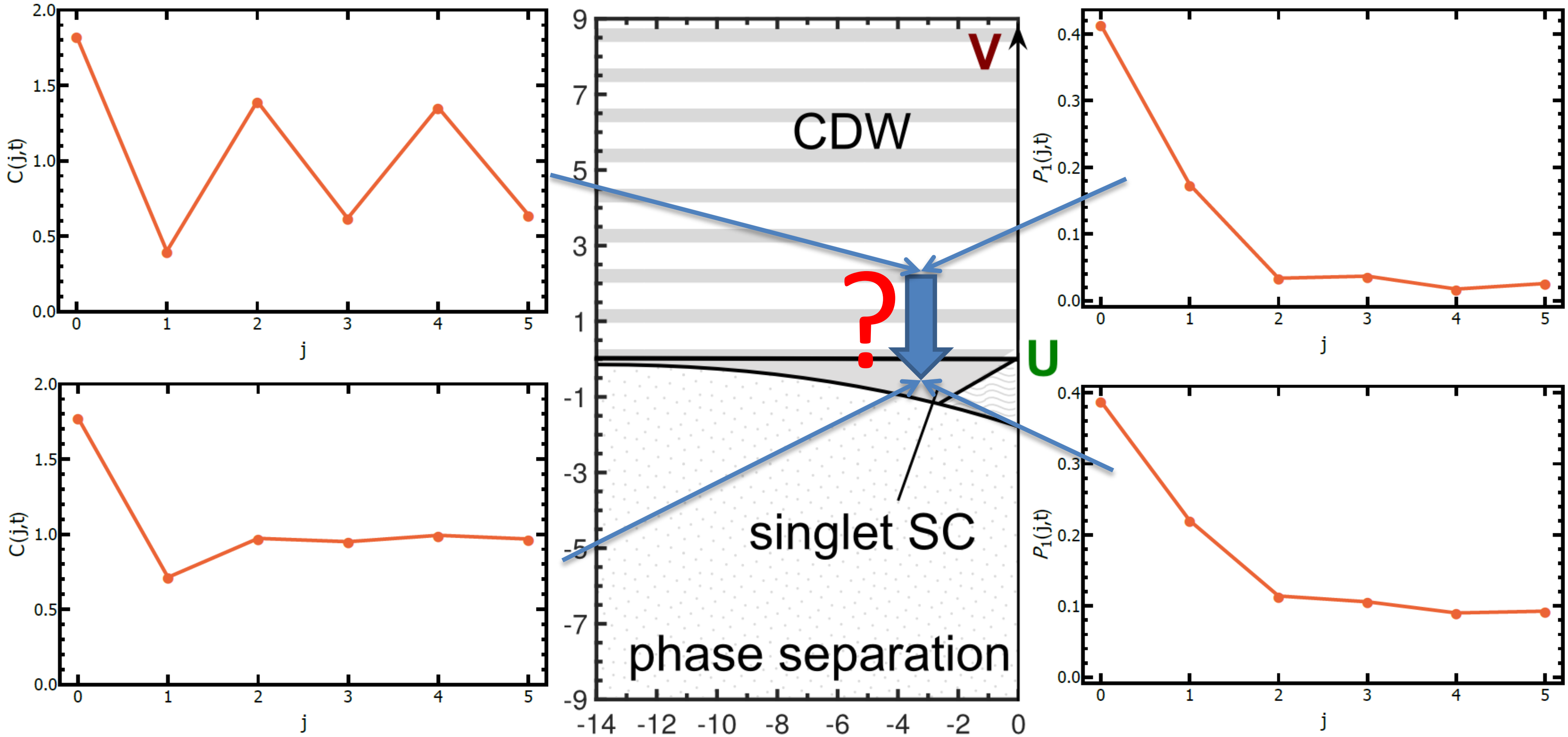
Density-density correlation function

$$C(j) = \frac{1}{L}\sum_{l=0}^{L-1}\langle\psi\,|\,\hat{n}_{l+j}\;\hat{n}_l\,|\,\psi\rangle$$

On-site correlation function

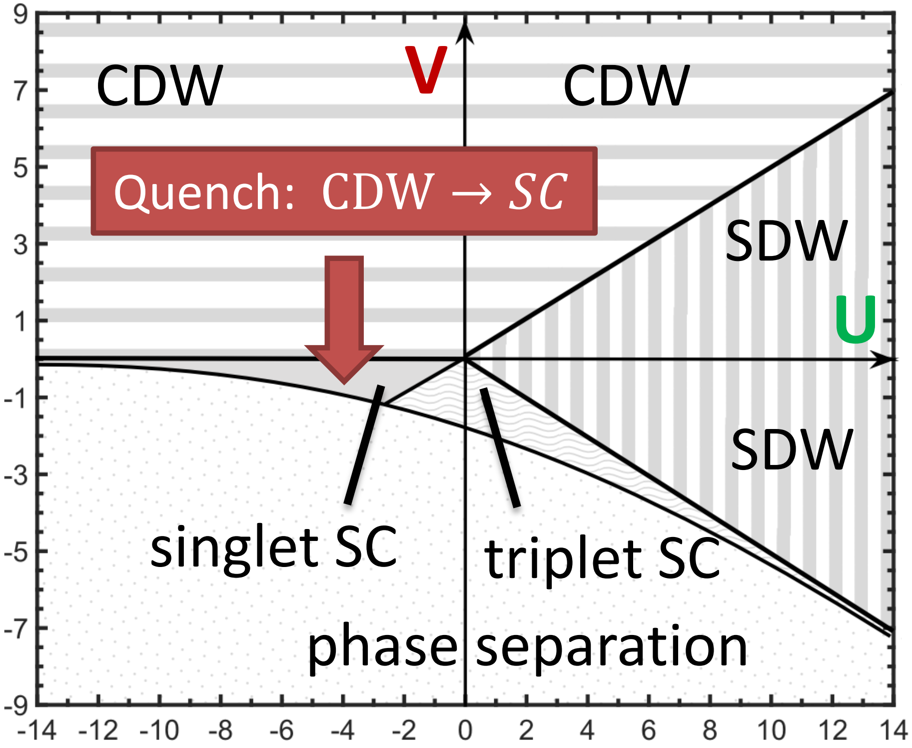
$$P_1(j) = \frac{1}{L}\sum_{l=0}^{L-1}\left\langle\psi\,\middle|\,\hat{c}^\dagger_{l+j\downarrow}\,\hat{c}^\dagger_{l+j\uparrow}\,\hat{c}_{l\uparrow}\,\hat{c}_{l\downarrow}\,\middle|\,\psi\right\rangle$$

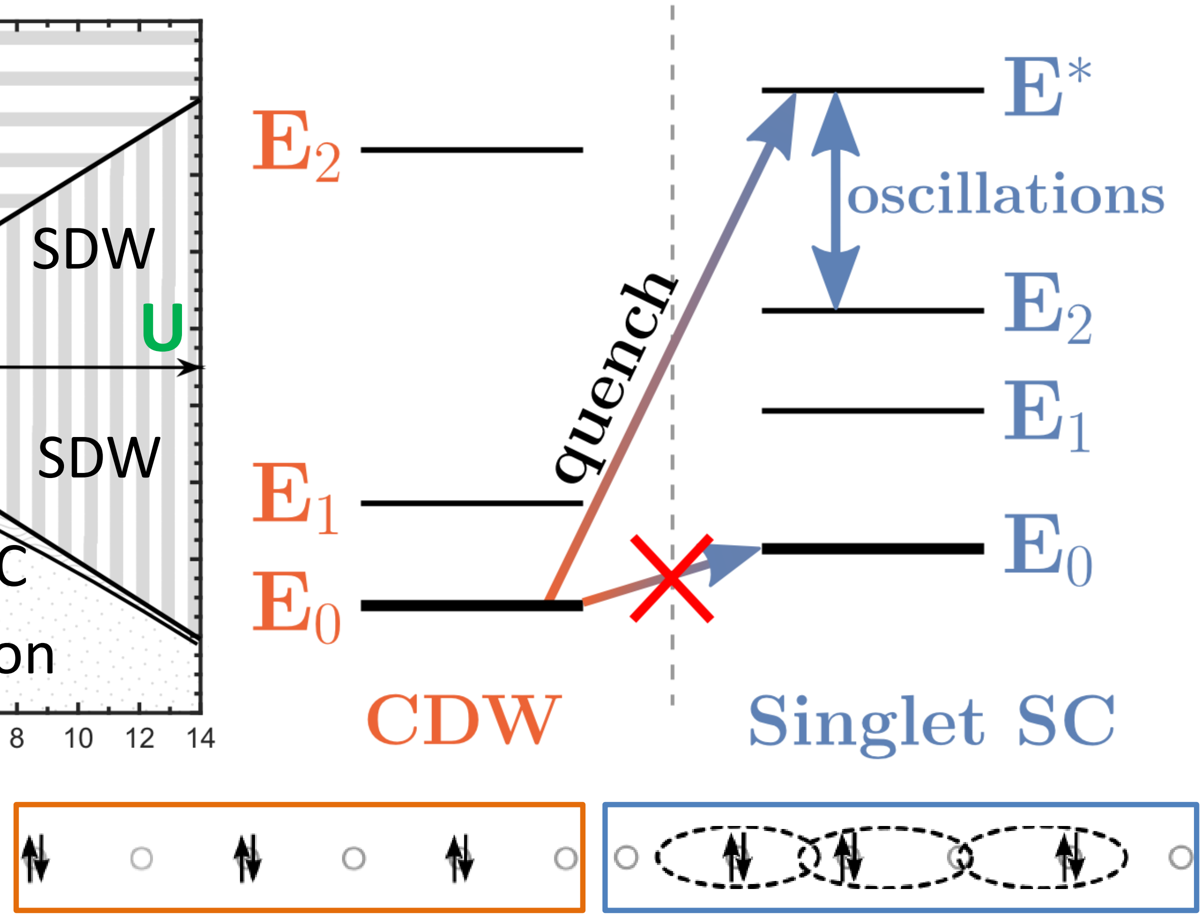


Results

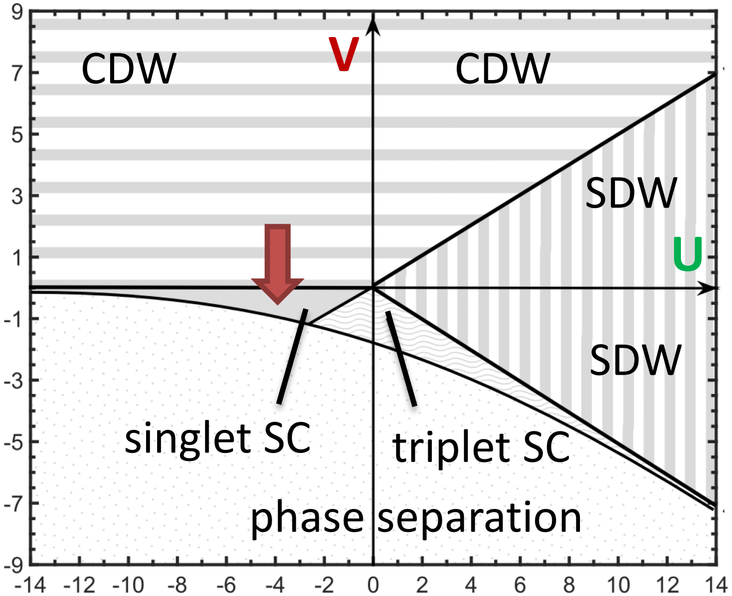
Interaction quench

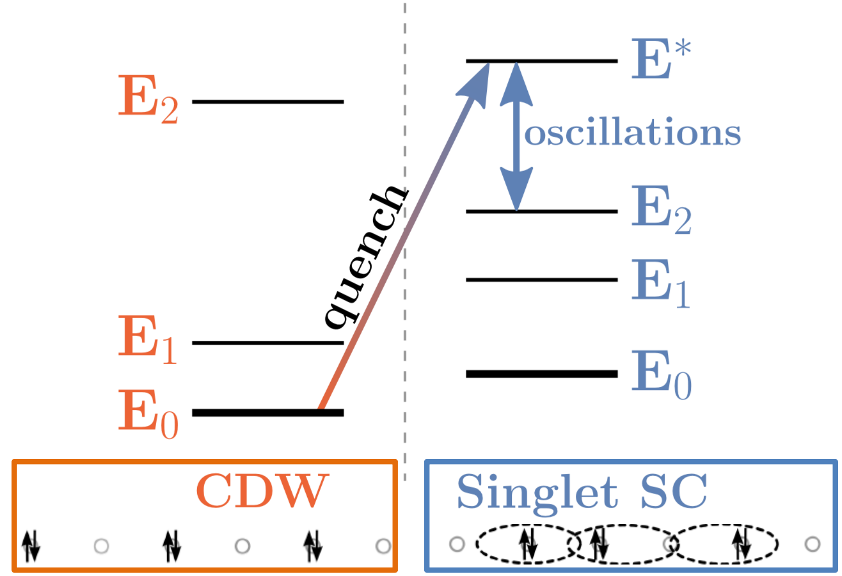
$$V \rightarrow V \cdot \Theta(t - t_0)$$

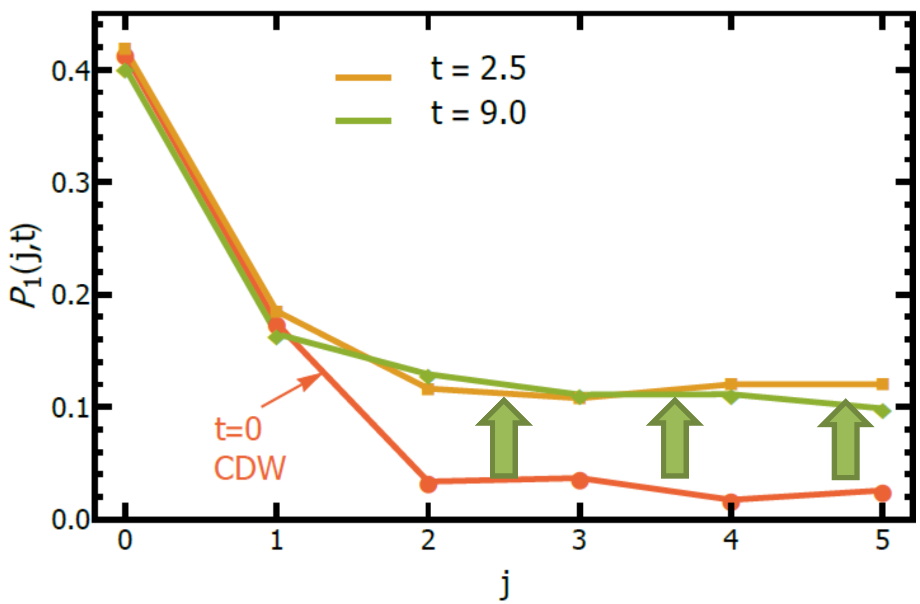


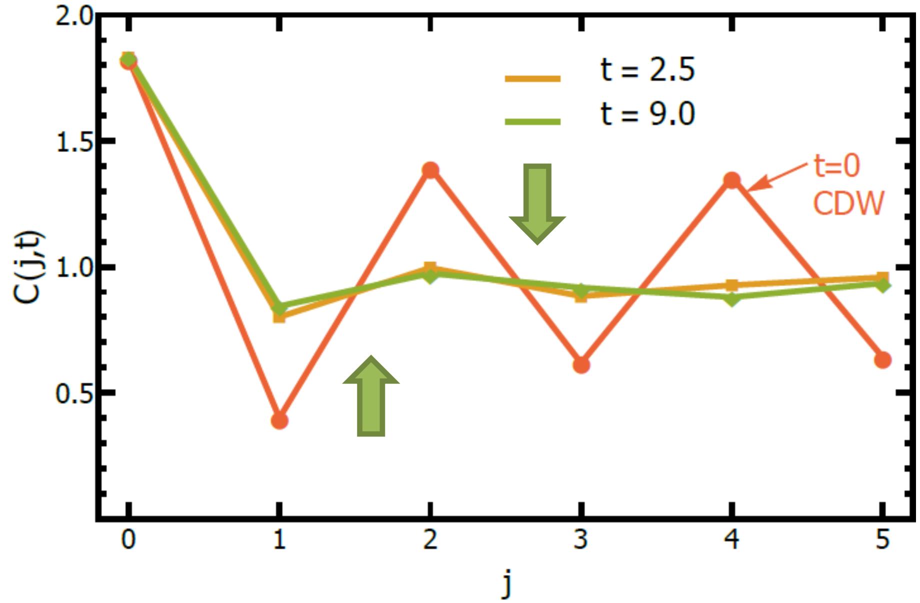


Induced SC? First indications









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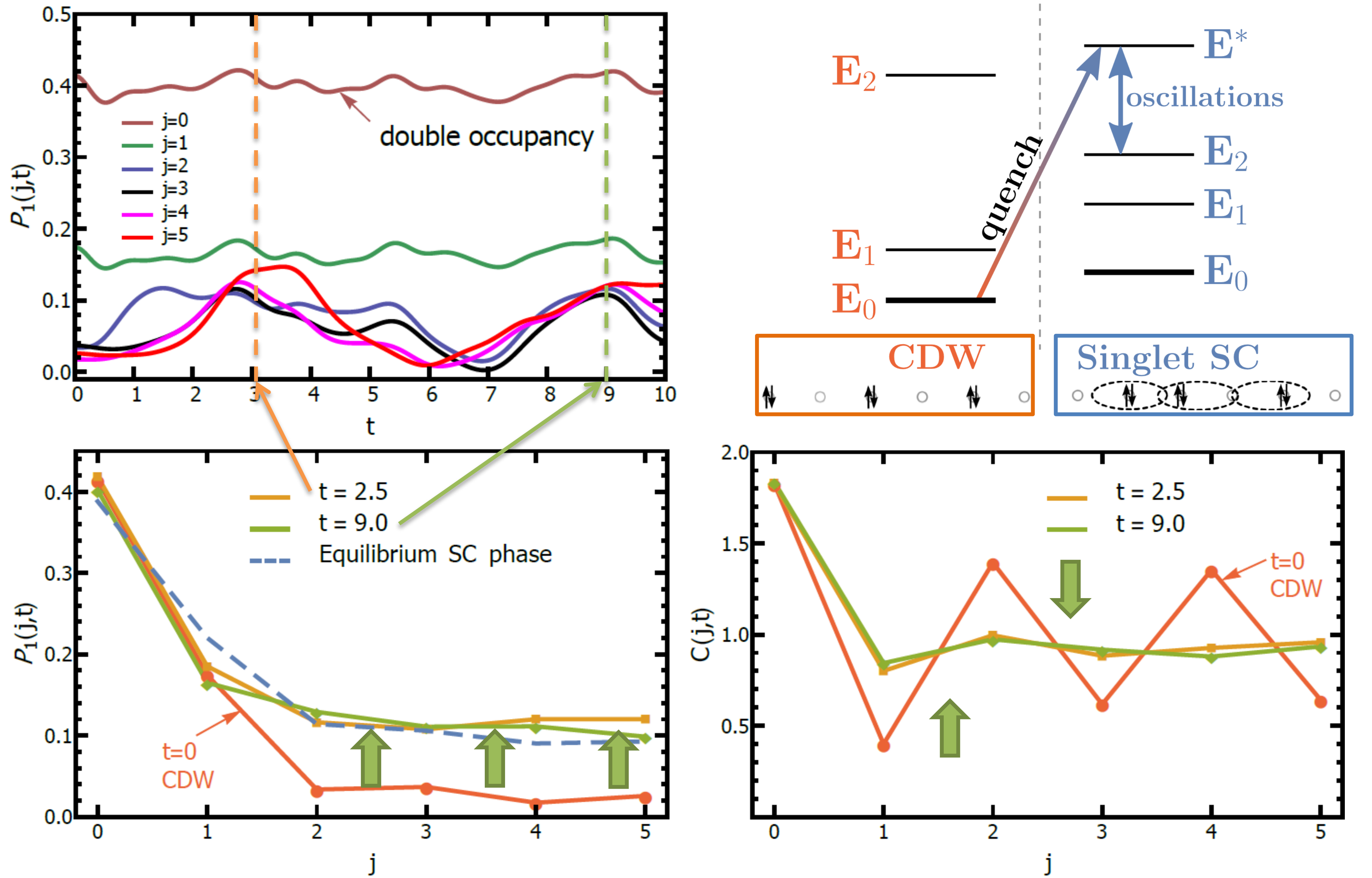
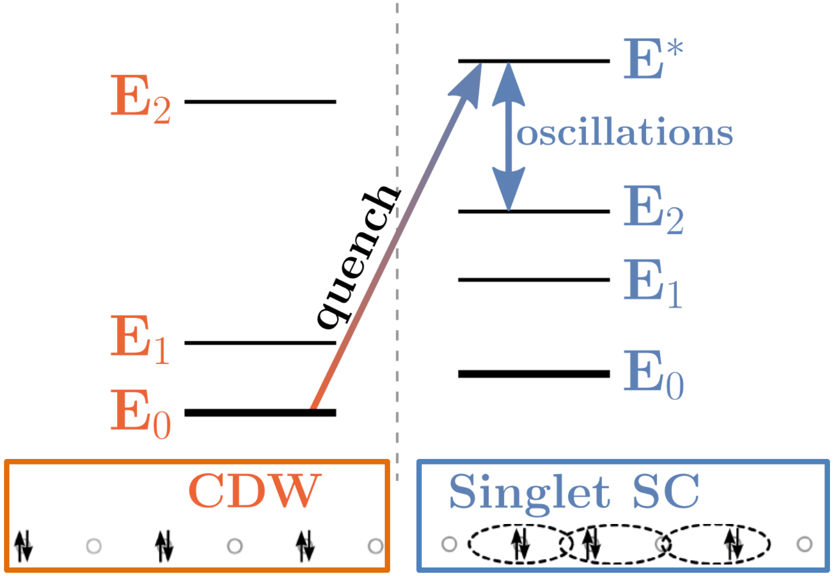
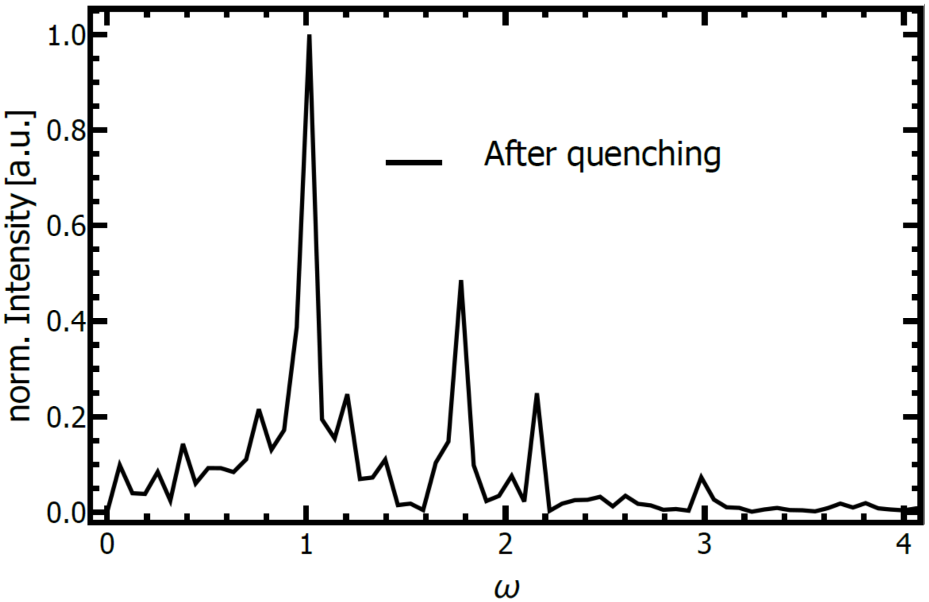


Transient Superconductivity



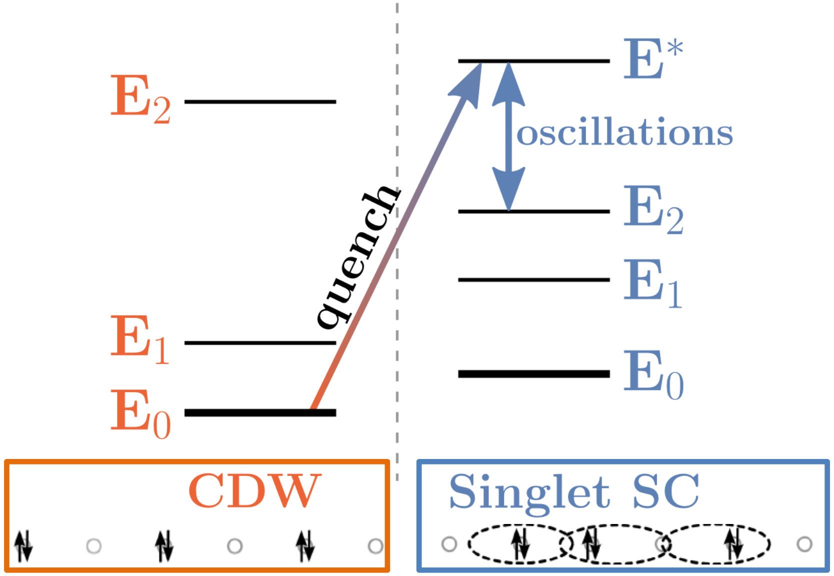


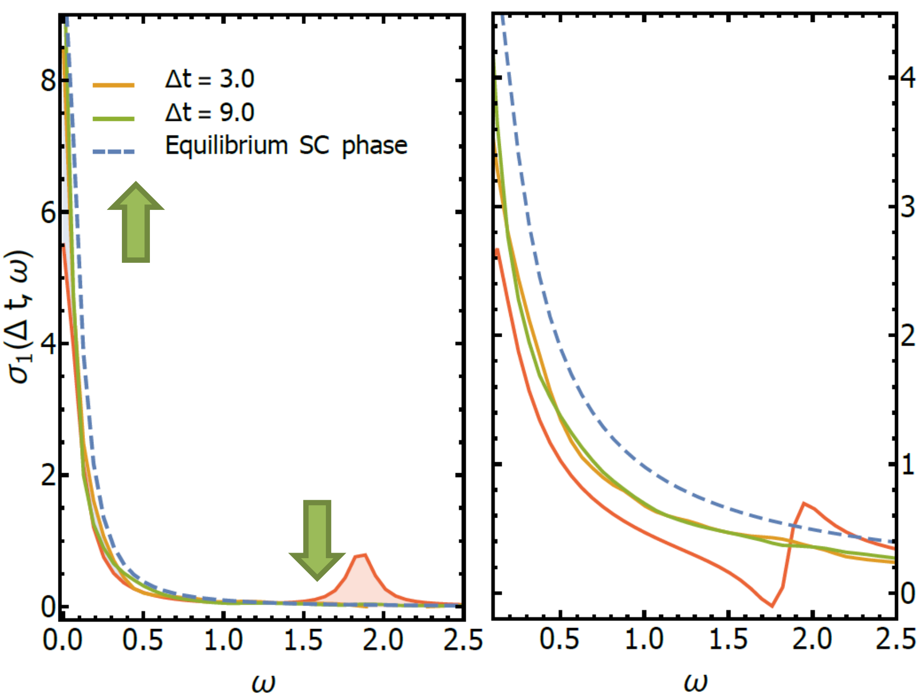
Transient Superconductivity

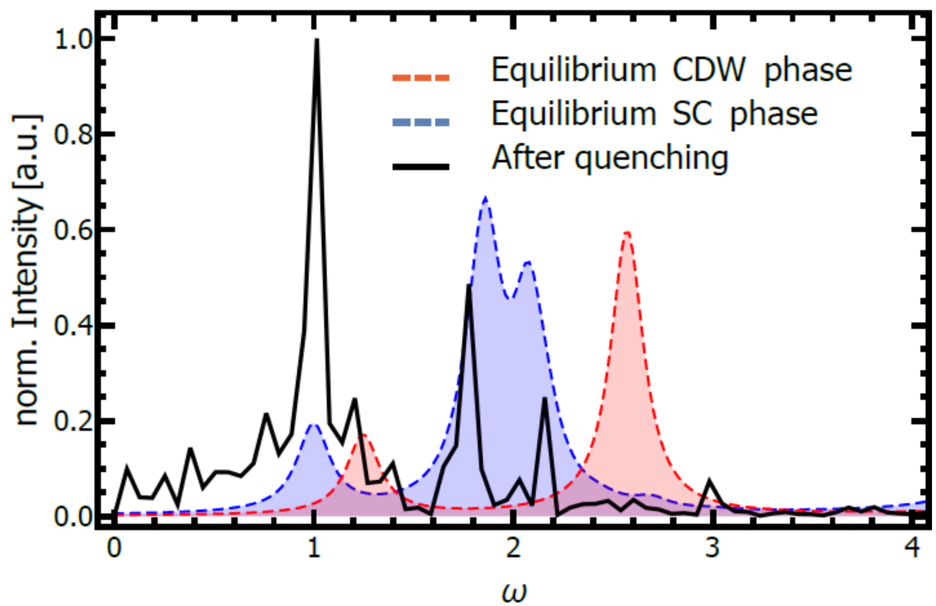
Transient Meissner effect ?

Time-dependent optical conductivity

$$\sigma(\Delta t,\omega) = \frac{j_{\mathrm{pr}}(\Delta t,\omega)}{i\,(\omega + i\,\eta) L\, A_{\mathrm{pr}}(\Delta t,\omega)}$$



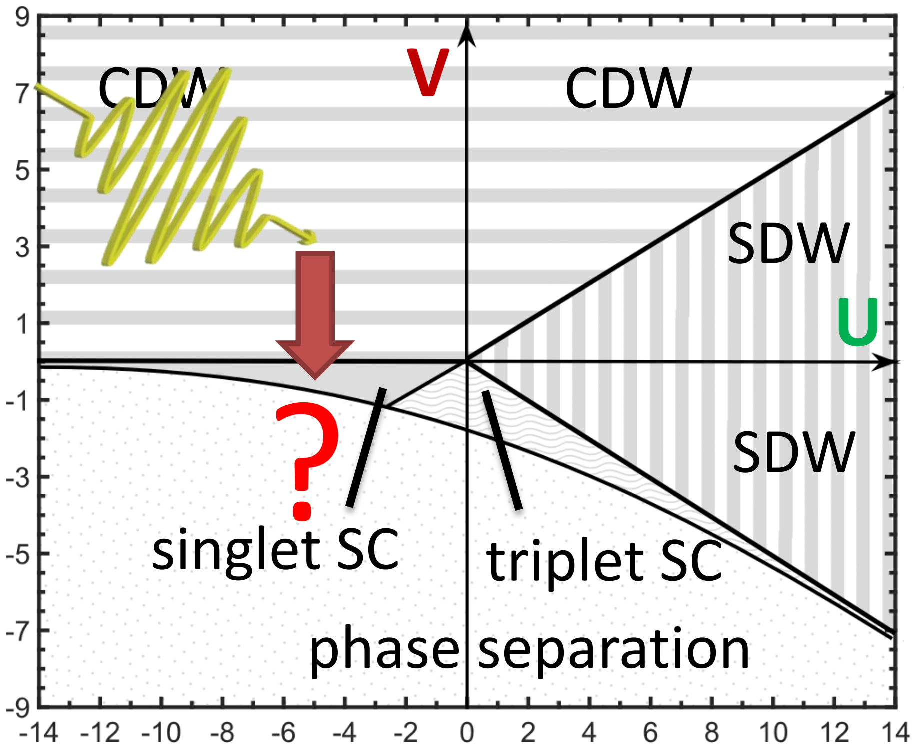


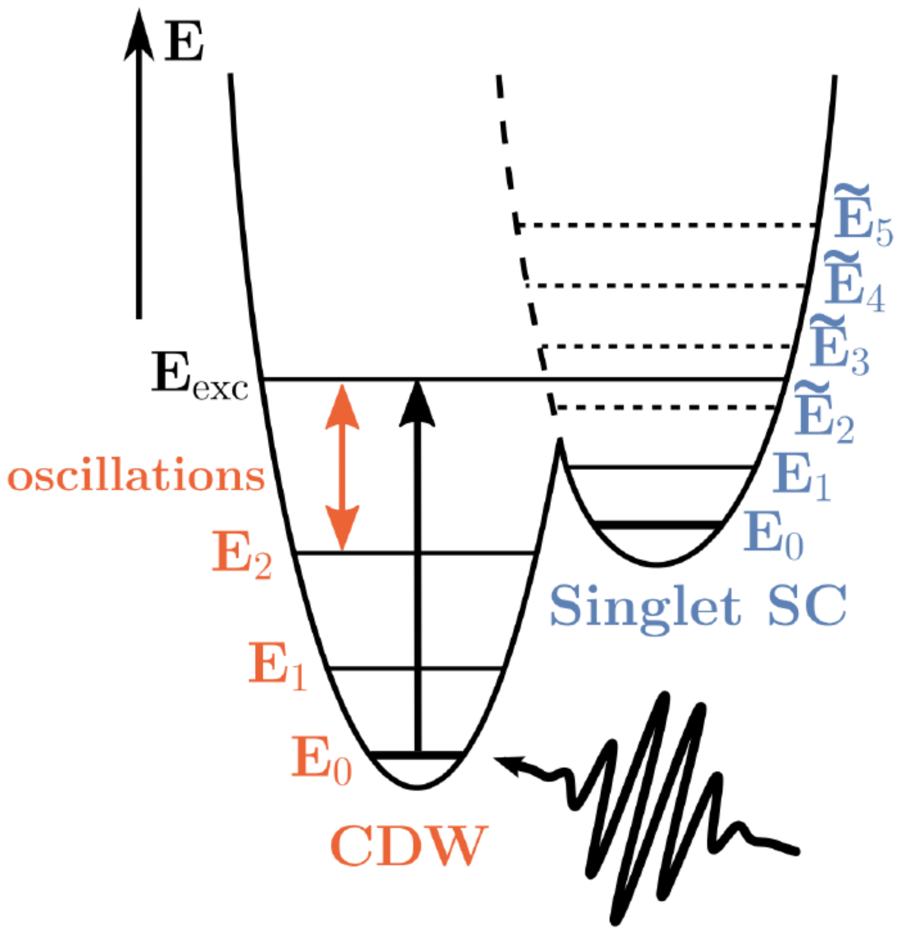


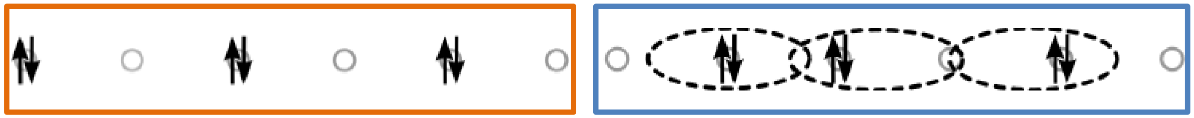
Pulse quench

$$t_h \rightarrow t_h \cdot e^{iA(t)}$$

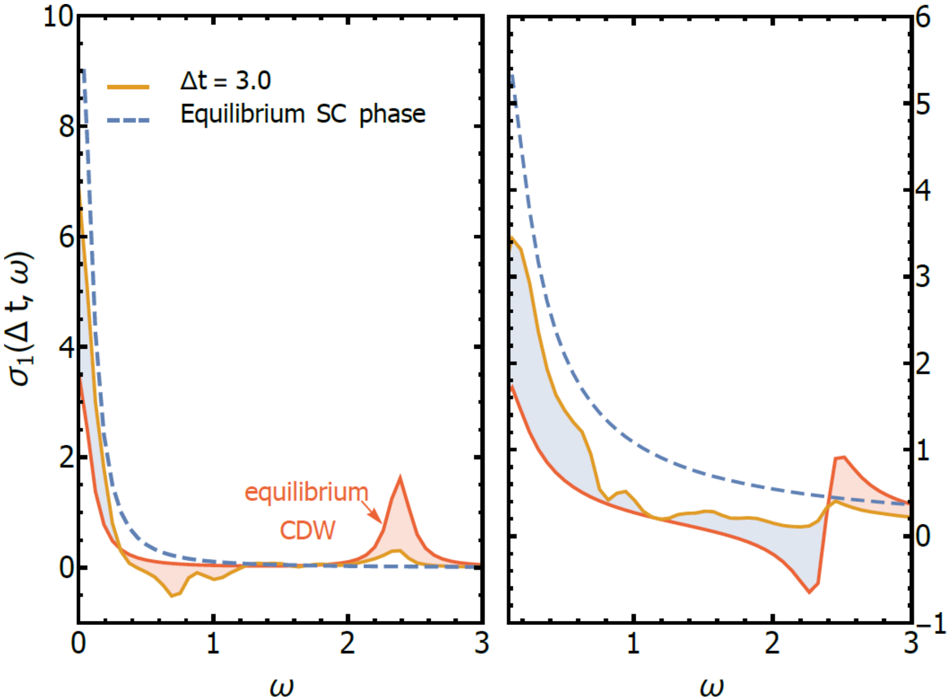
$$A(t) = A_0 e^{-(t-t_0)^2/2\tau^2} \cos[\omega_0(t-t_0)]$$





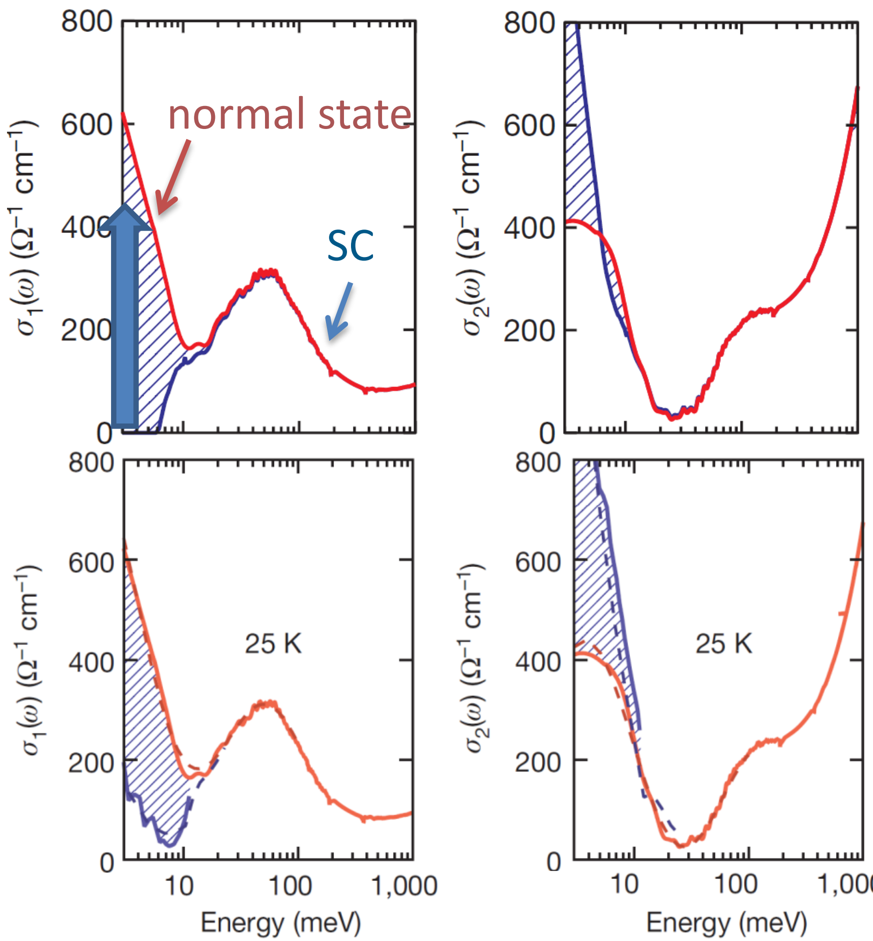


Optical conductivity

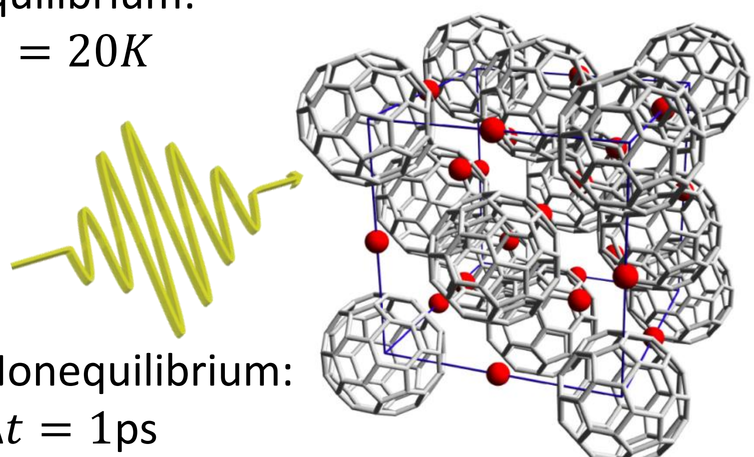


s-wave superconductor K_3C_{60}

*Mitrano et.al, Nature **530** (2016)*

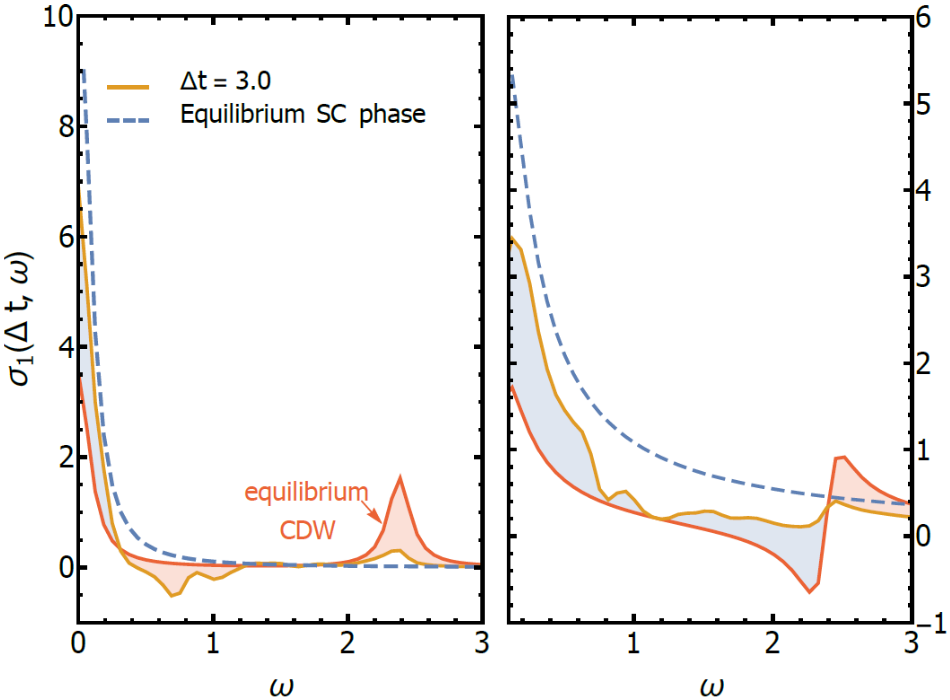


Equilibrium:
$T_c = 20K$



Nonequilibrium:
$\Delta t = 1\text{ps}$

Optical conductivity



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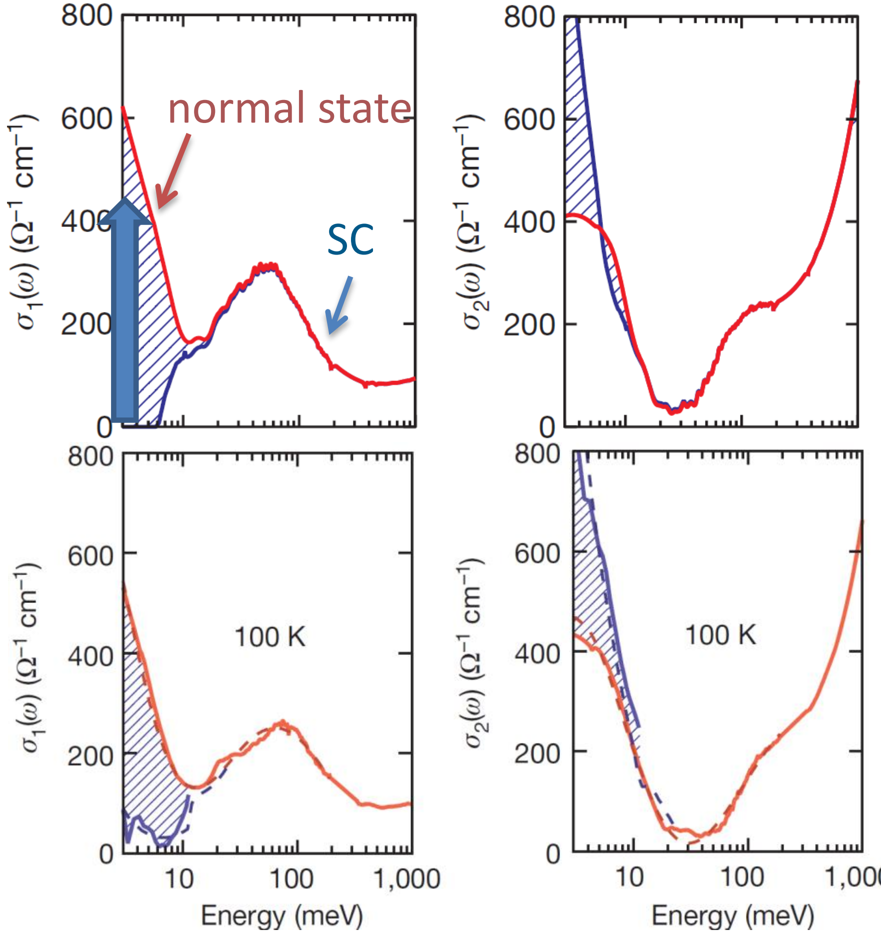

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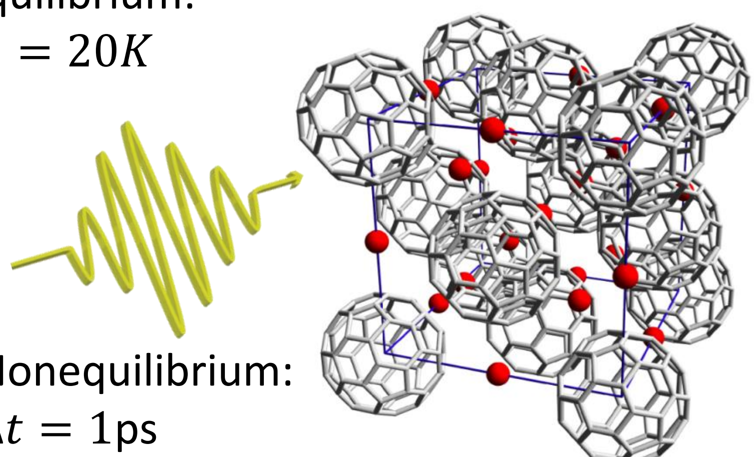

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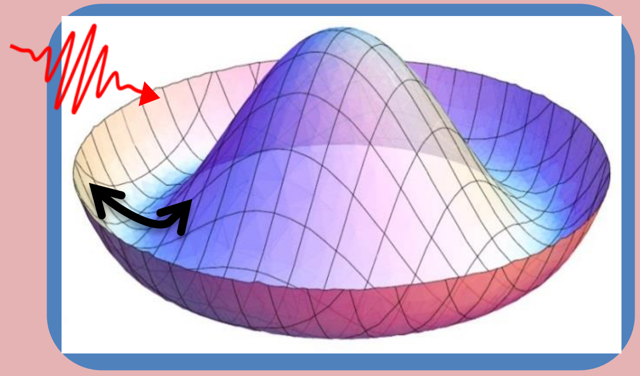
Outline

New! Higgs spectroscopy



» Direct detection of the SC gap

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Order parameter oscillations in Superconductors

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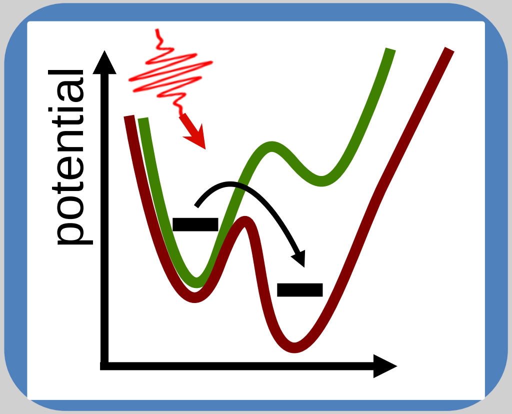
optical control



» New transient ground state

Part 1

Induced Superconductivity in the extended Hubbard Model

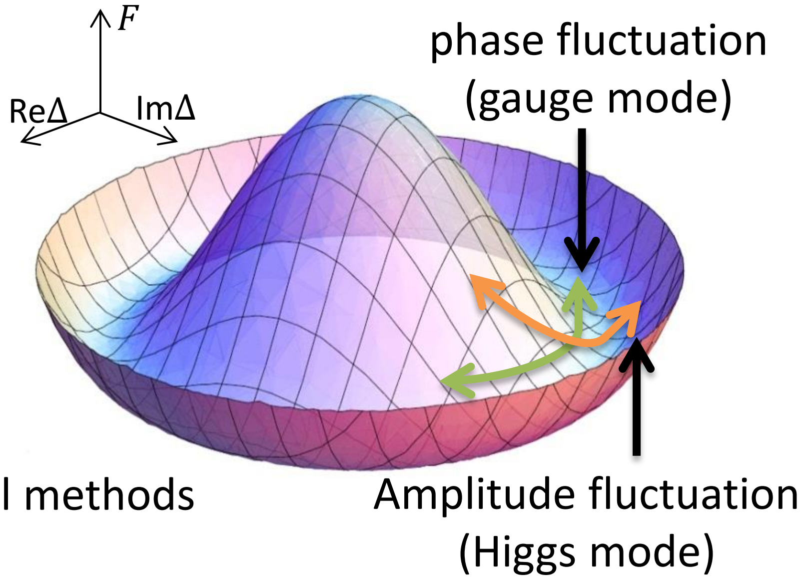
N. Bittner, T. Tohyama, and D. Manske, in preparation

Collective modes in nutshell

One-band superconductor

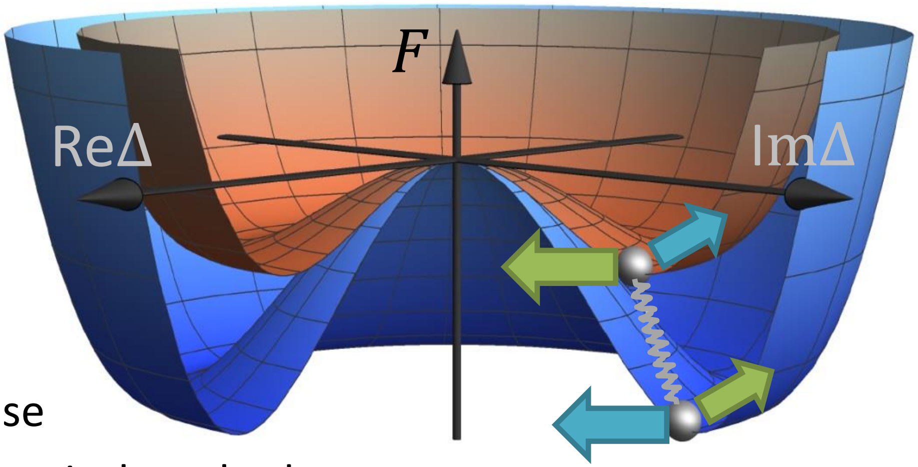
- **Phase (gauge) mode**
  - ⇨ Shift to plasma frequency (**Anderson-Higgs mechanism**)
    P. W. Anderson (1963), P. W. Higgs (1964)
- **Amplitude (Higgs) mode**
  - ⇨ **lies at 2Δ**. Not detectable in Raman response
  - ⇨ **No charge or dipole moment.** Not seen by optical methods
  - ⇨ indirect detection in **special cases** possible

M.-A. Measson et al., PR B ***89****, 060503 (2014), D. Sherman et al., Nat. Phys.* ***11****, 188 (2015)*



Introduction: Collective modes in nutshell

One-band superconductor



» Phase (gauge) mode

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» Amplitude (Higgs) mode

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⇨ indirect detection in **special cases** possible

*M.-A. Measson et al., PR B **89**, 060503 (2014), D. Sherman et al., Nat. Phys. **11**, 188 (2015)*

Two-band superconductor

$$\omega_L^2 = 4\,|\Delta_1|\,|\Delta_2|\;\frac{V}{V_{22} - V_{11}V^2}$$

*A. J. Leggett, Prog. Theor. Phys. **36**, 901 (1966)*

» **Leggett mode** = out-of-phase vibration of the superconducting condensates

⇨ **has no analogy** in the one band case

⇨ **intrinsic Josephson effect**

⇨ Experimental **evidence in Raman response** for MgB_2

*G. Blumberg et al., Phys. Rev. Lett. **99**, 227002 (2007)*

→ Direct detection of the Higgs mode in a superconductor?

→ Is there a Josephson effect in nonequilibrium?

Method

Shake the condensate! ⇒

1. **THz pulse**
2. **short pulse width (non-adiabatic regime)**
3. **low pulse fluency**

Microscopic approach:

» Hamiltonian for a two-band superconductor:

$$\hat{H} = \sum_{l=1}^{2} \left[\sum_{\boldsymbol{k}\sigma} \epsilon_{\boldsymbol{k}l} \, \hat{c}^{\dagger}_{\boldsymbol{k}\sigma l} \hat{c}_{\boldsymbol{k}\sigma l} + \sum_{\boldsymbol{k}} \left(\Delta_l \quad \hat{c}^{\dagger}_{\boldsymbol{k}\uparrow l} \hat{c}^{\dagger}_{-\boldsymbol{k}\downarrow l} + h.c. \right) \right] \leftarrow \text{BCS}$$

» Assumptions:

(a) tetragonal lattice

(b) *s*-wave order parameters

» Energy gap equations:

$$\Delta_1 = \sum_{\boldsymbol{k}} [V_{11} \langle \hat{c}_{-\boldsymbol{k}\downarrow 1} \hat{c}_{\boldsymbol{k}\uparrow 1} \rangle \quad + V \langle \hat{c}_{-\boldsymbol{k}\downarrow 2} \hat{c}_{\boldsymbol{k}\uparrow 2} \rangle \quad V_{11}]$$

$$\Delta_2 = \sum_{\boldsymbol{k}} [V_{22} \langle \hat{c}_{-\boldsymbol{k}\downarrow 2} \hat{c}_{\boldsymbol{k}\uparrow 2} \rangle \quad + V \langle \hat{c}_{-\boldsymbol{k}\downarrow 1} \hat{c}_{\boldsymbol{k}\uparrow 1} \rangle \quad V_{11}] \qquad \text{with } V = V_{12}/V_{11}$$

Microscopic approach:

» Hamiltonian for a two-band superconductor **in nonequilibrium**:

$$\hat{H}(t) = \sum_{l=1}^{2} \left[\sum_{\boldsymbol{k}\sigma} \epsilon_{\boldsymbol{k}l}\, \hat{c}^\dagger_{\boldsymbol{k}\sigma l}\hat{c}_{\boldsymbol{k}\sigma l} + \sum_{\boldsymbol{k}} \left(\Delta_l(t)\, \hat{c}^\dagger_{\boldsymbol{k}\uparrow l}\hat{c}^\dagger_{-\boldsymbol{k}\downarrow l} + h.c. \right) \right] \leftarrow \text{BCS}$$

$$-e\hbar \sum_{\boldsymbol{kq}\sigma l} \frac{(2\boldsymbol{k}+\boldsymbol{q}) \cdot \boldsymbol{A}_{\boldsymbol{q}}(t)}{2m_l}\, \hat{c}^\dagger_{\boldsymbol{k}+\boldsymbol{q}\sigma l}\hat{c}_{\boldsymbol{k}\sigma l} + e^2 \sum_{\boldsymbol{kq}\sigma l} \frac{\sum_{\boldsymbol{q}'} \boldsymbol{A}_{\boldsymbol{q}-\boldsymbol{q}'}(t) \cdot \boldsymbol{A}_{\boldsymbol{q}'}(t)}{2m_l}\, \hat{c}^\dagger_{\boldsymbol{k}+\boldsymbol{q}\sigma l}\hat{c}_{\boldsymbol{k}\sigma l}$$

probe pulse

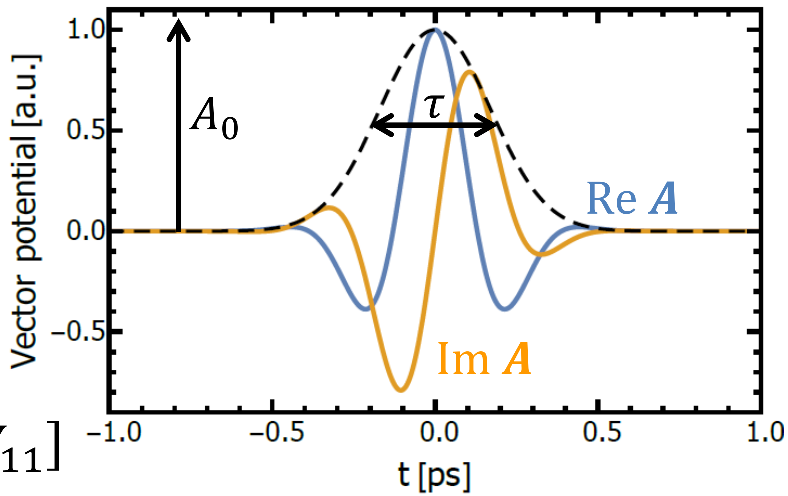
pump pulse

coupling to the field

» Assumptions:

(a) tetragonal lattice

(b) *s*-wave order parameters



» Energy gap equations:

$$\Delta_1(t) = \sum_{\boldsymbol{k}} [V_{11}\langle \hat{c}_{-\boldsymbol{k}\downarrow 1}\hat{c}_{\boldsymbol{k}\uparrow 1}\rangle(t) + V\langle \hat{c}_{-\boldsymbol{k}\downarrow 2}\hat{c}_{\boldsymbol{k}\uparrow 2}\rangle(t) V_{11}]$$

$$\Delta_2(t) = \sum_{\boldsymbol{k}} [V_{22}\langle \hat{c}_{-\boldsymbol{k}\downarrow 2}\hat{c}_{\boldsymbol{k}\uparrow 2}\rangle(t) + V\langle \hat{c}_{-\boldsymbol{k}\downarrow 1}\hat{c}_{\boldsymbol{k}\uparrow 1}\rangle(t) V_{11}]$$

with $V = V_{12}/V_{11}$

Density Matrix Theory

» Bogoliubov-Valatin transformation (superconducting state):

$$\alpha^\dagger_{\mathbf{k}l} = u_{\mathbf{k}l}\hat{c}^\dagger_{\mathbf{k}\uparrow l} + v^*_{\mathbf{k}l}\hat{c}_{-\mathbf{k}\downarrow l} \qquad \beta^\dagger_{\mathbf{k}l} = u_{\mathbf{k}l}\hat{c}^\dagger_{-\mathbf{k}\downarrow l} - v^*_{\mathbf{k}l}\hat{c}_{\mathbf{k}\uparrow l}$$

$\Rightarrow$ energy gap equation:

$$\Delta_l(t) = \sum_{\mathbf{k}j} V_{lj}\left(u_{\mathbf{k}j}v_{\mathbf{k}j}\left[1 - \langle\alpha^\dagger_{\mathbf{k}j}\alpha_{\mathbf{k}j}\rangle - \langle\beta^\dagger_{\mathbf{k}j}\beta_{\mathbf{k}j}\rangle\right] + u^2_{\mathbf{k}j}\langle\beta_{\mathbf{k}j}\alpha_{\mathbf{k}j}\rangle - v^2_{\mathbf{k}j}\langle\alpha^\dagger_{\mathbf{k}j}\beta^\dagger_{\mathbf{k}j}\rangle\right)(t)$$

» Derivation of the equations of motions for the Bogoliubov quasiparticle densities:

$$\langle\alpha^\dagger_{\mathbf{k}l}\alpha_{\mathbf{k}l}\rangle(t), \qquad \langle\beta^\dagger_{\mathbf{k}l}\beta_{\mathbf{k}l}\rangle(t), \qquad \langle\beta_{\mathbf{k}l}\alpha_{\mathbf{k}l}\rangle(t), \qquad \langle\alpha^\dagger_{\mathbf{k}l}\beta^\dagger_{\mathbf{k}l}\rangle(t)$$

» Density matrix theory:

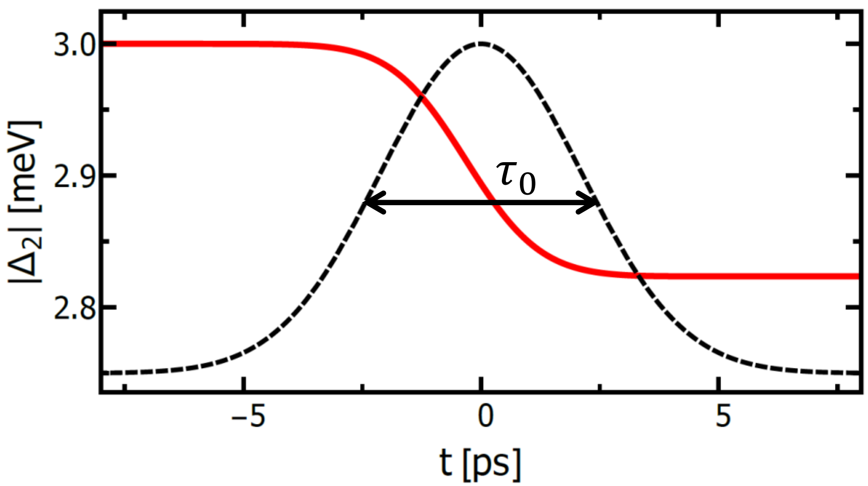
$$\frac{d}{dt}\left(\hat{c}^\dagger_{\mathbf{k}_1 l}\hat{c}_{\mathbf{k}_2 l}\right) = \frac{i}{\hbar}\left[\widehat{H}, \hat{c}^\dagger_{\mathbf{k}_1 l}\hat{c}_{\mathbf{k}_2 l}\right] + \frac{\partial}{\partial t}\left(\hat{c}^\dagger_{\mathbf{k}_1 l}\hat{c}_{\mathbf{k}_2 l}\right)$$

$\Rightarrow$ yield equations of motions for the expectation values

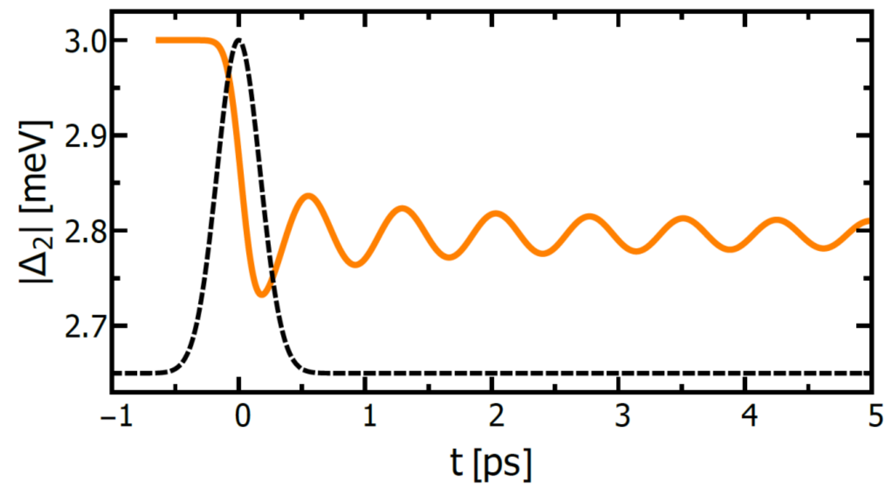
» Numerical solution by using a Runge-Kutta method

Results: two independent bands
(single band solution)

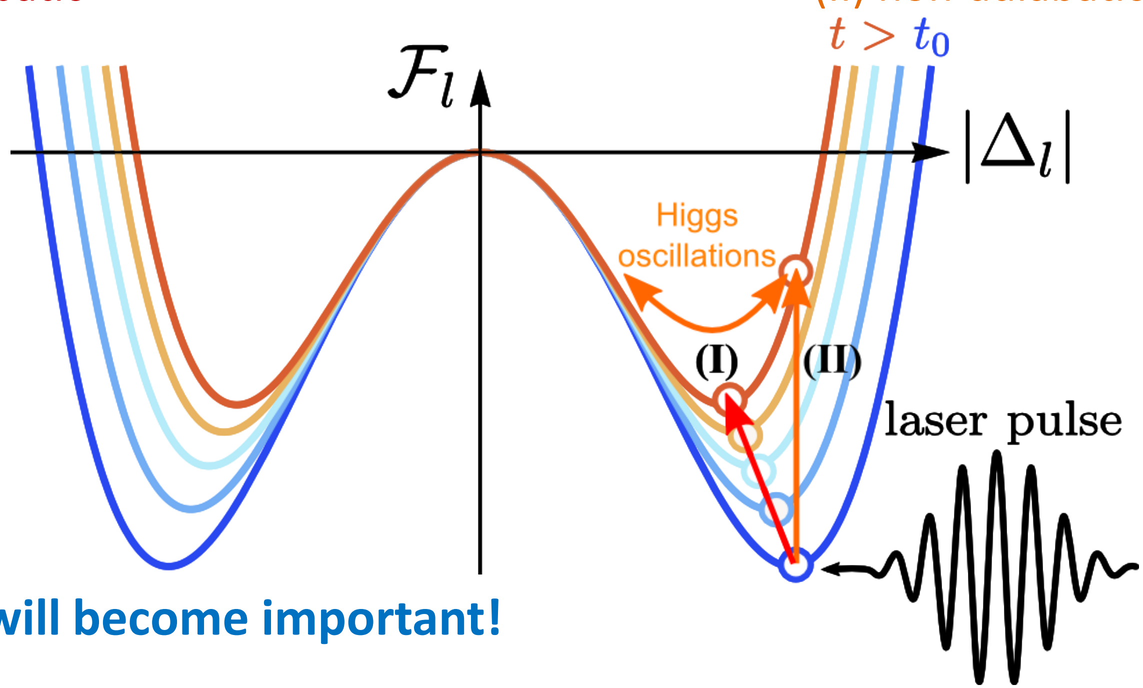
Higgs excitation in nonequilibrium



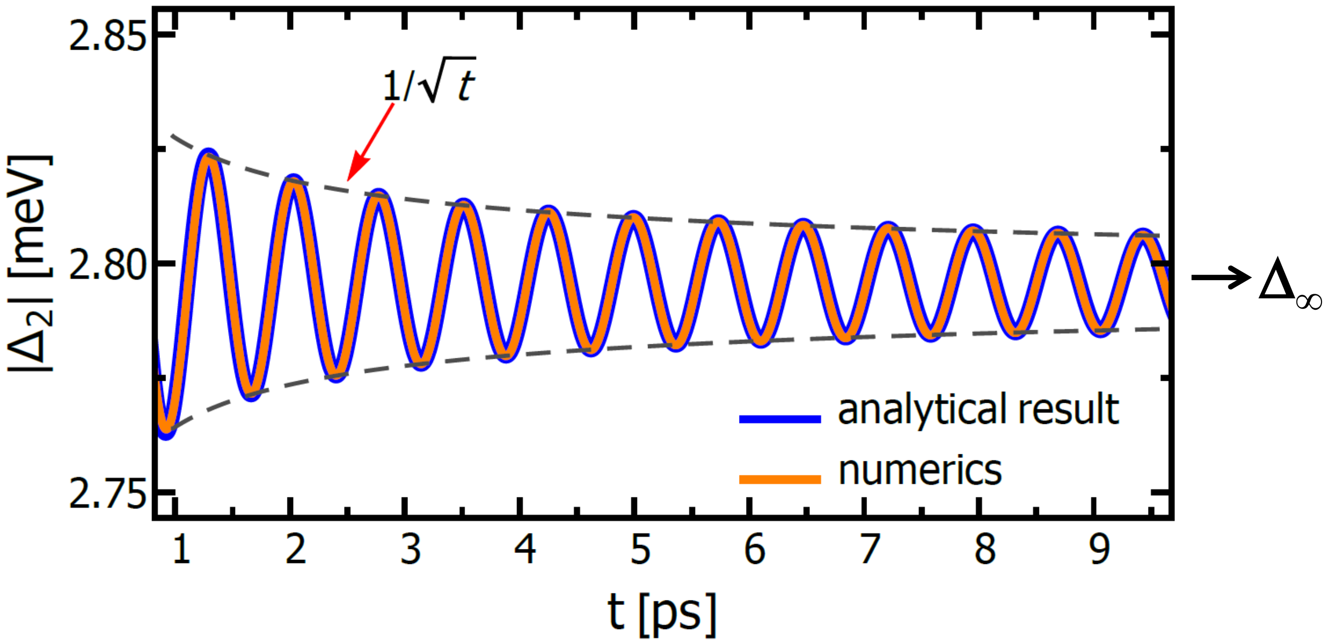
(I) adiabatic

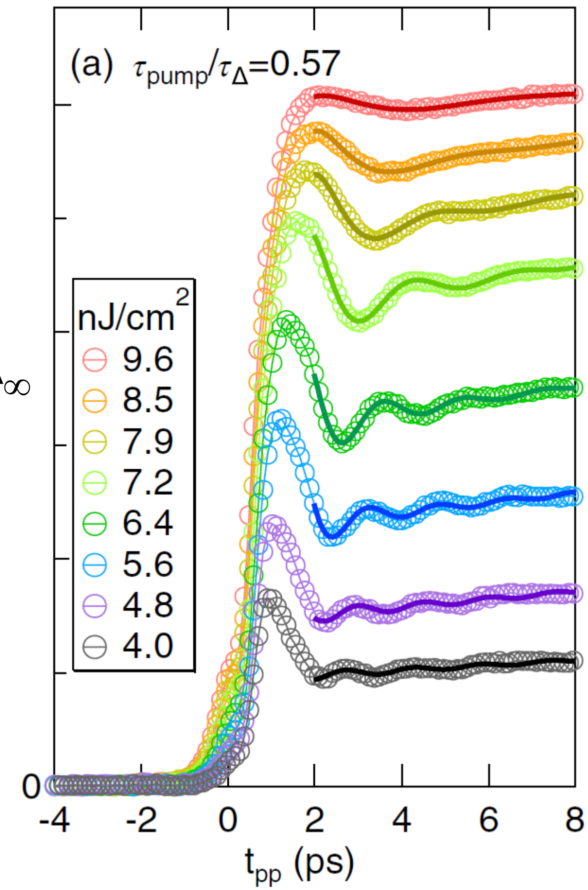


(II) non-adiabatic



- **Gaussian form of the laser pulses**
- **Pulse duration τ_0 will become important!**





Matsunaga et al., PRL 2013

» Analytical expression (**interaction quench**):

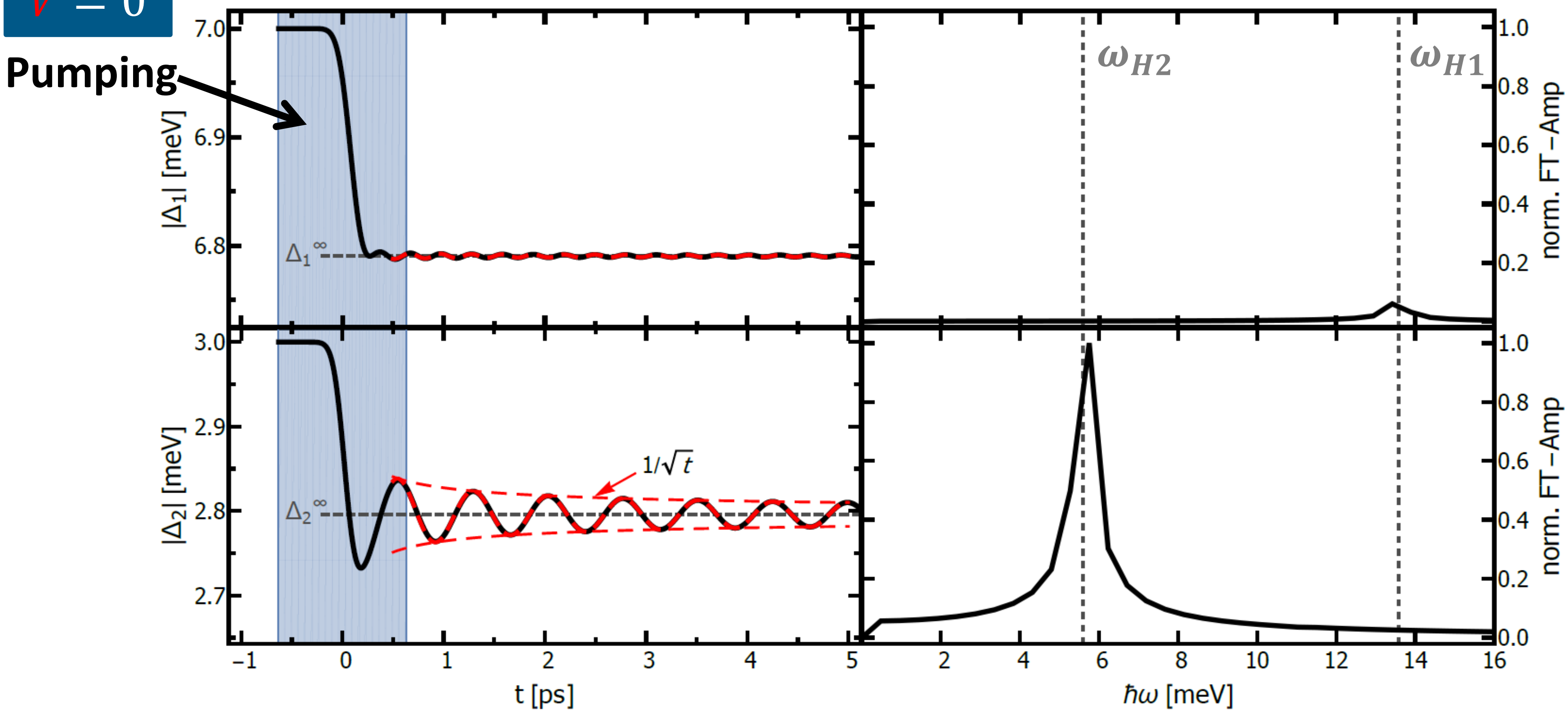
$$\frac{|\Delta|}{|\Delta_\infty|} = 1 + \Gamma \frac{\cos\left(\frac{2|\Delta_\infty|}{\hbar} t + \Phi\right)}{\sqrt{t\Delta_\infty}}$$

Yuzbashyan, Tsyplyatyev, Altshuler, PRL 2006

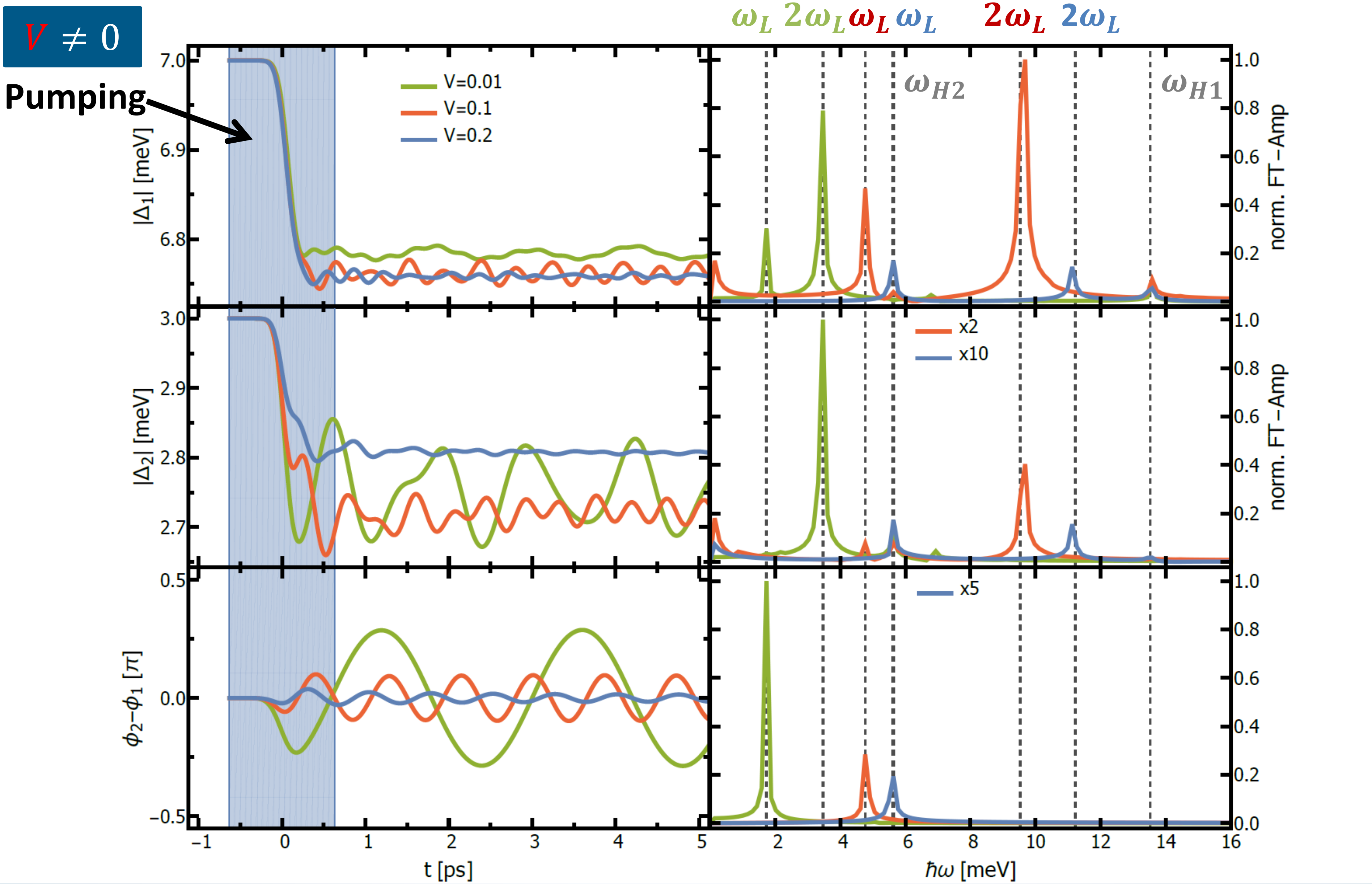
Results: two coupled bands
(Josephson effect in nonequilibrium)

Leggett mode in nonequilibrium?

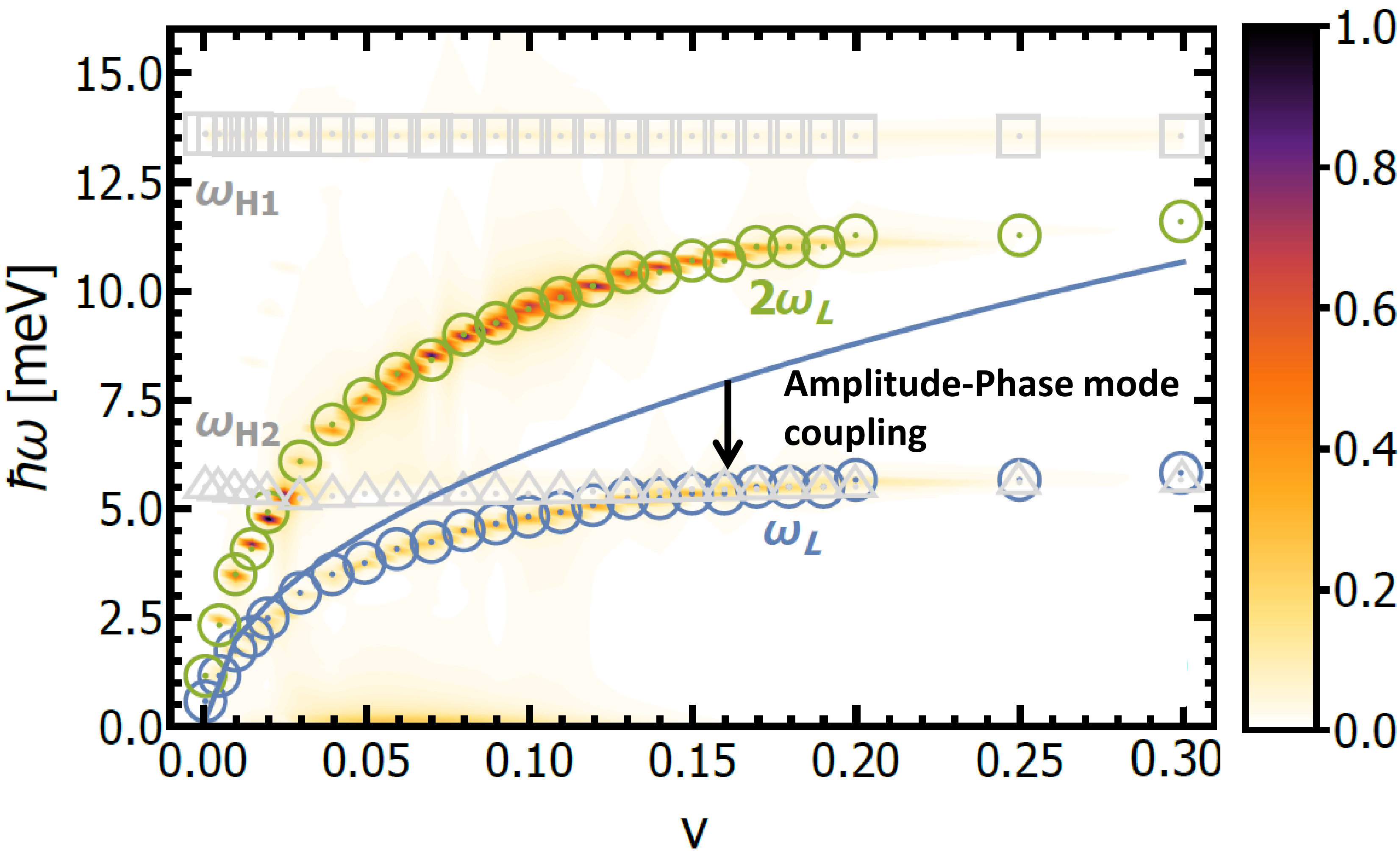
$V = 0$



Leggett mode in nonequilibrium?



New Legget mode in nonequilibrium: prediction

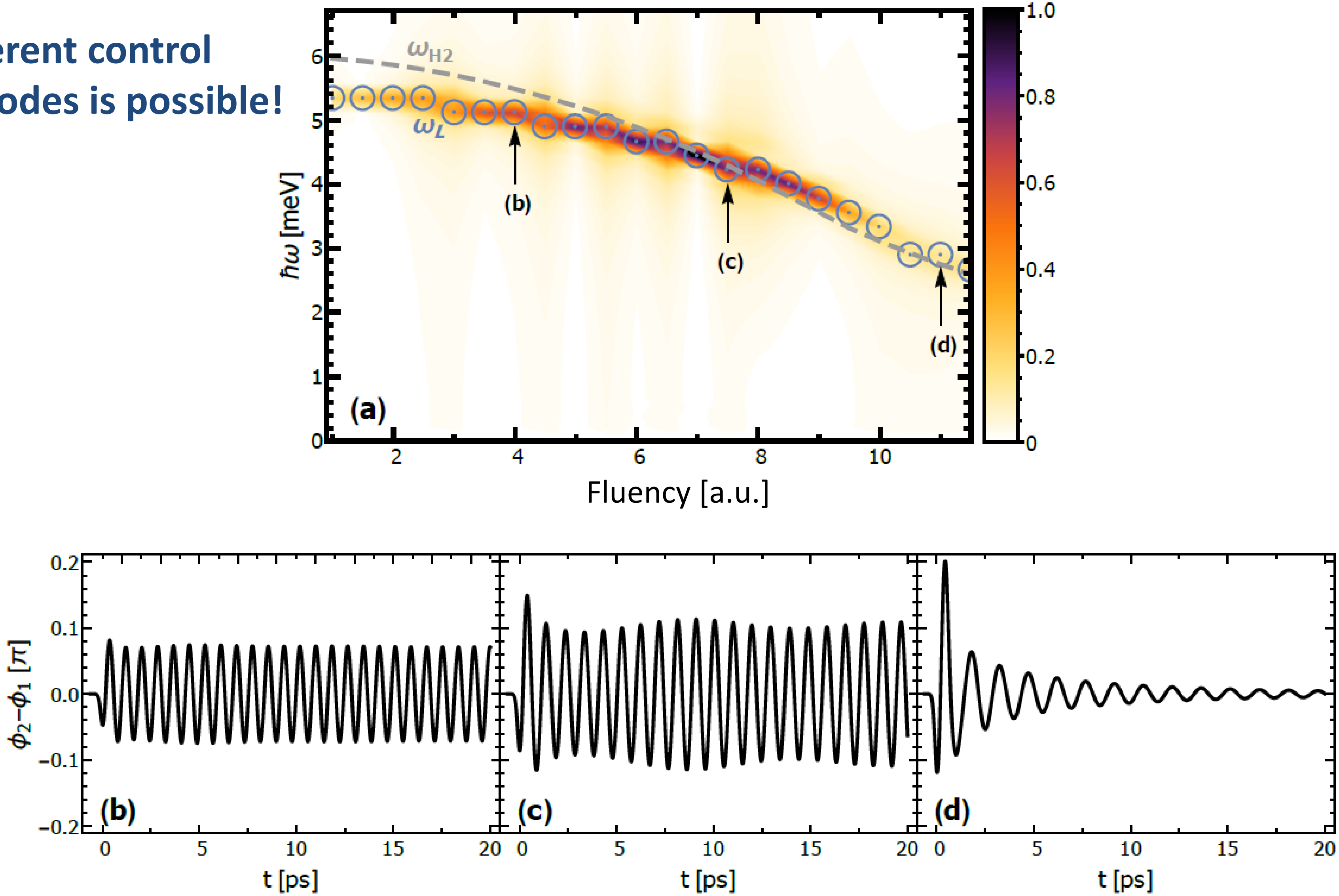


» **Amplitude-Phase mode coupling in nonequilibrium**

» **Amplitude-channel: harmonic $2\omega_L$ only present in nonequilibrium**

Intensity dependence

Coherent control of modes is possible!

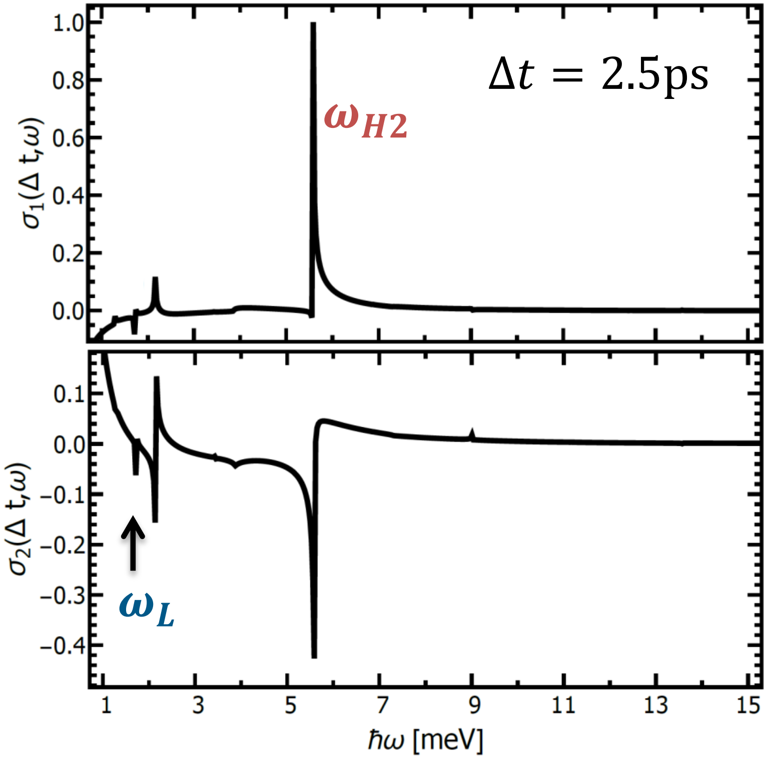


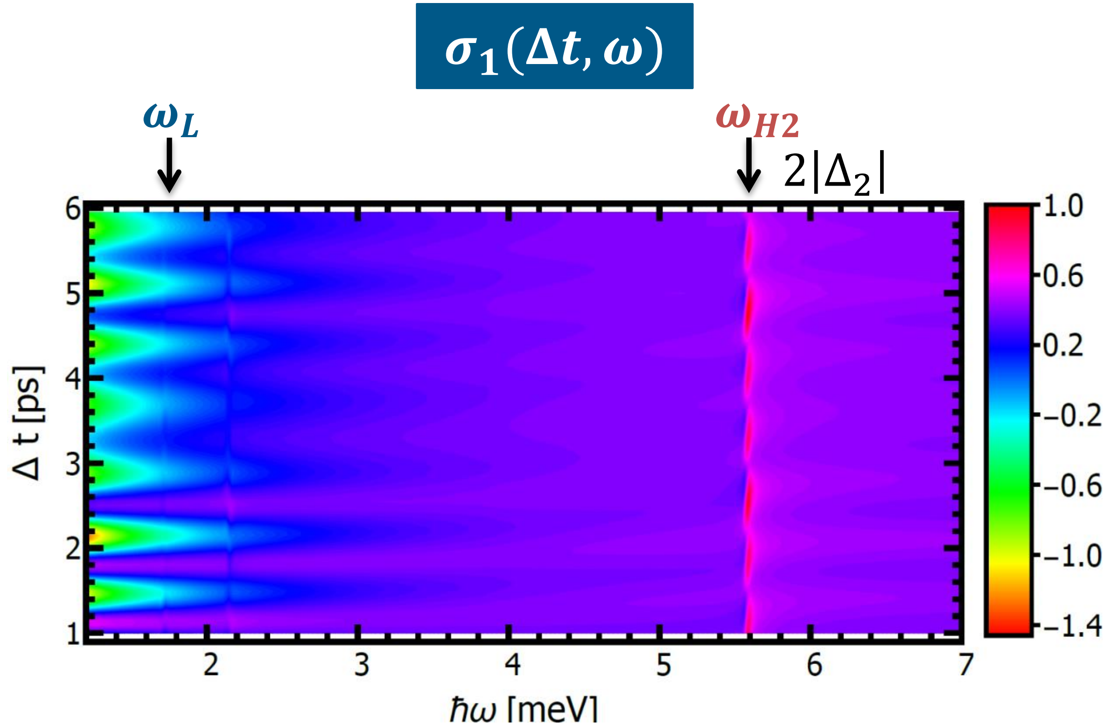
Optical conductivity (v=0.01)

» Pump-probe optical conductivity shows signatures of **non-adiabatic dynamics**

$$\sigma(\Delta t,\omega) = j(\Delta t,\omega)/\left[i\omega A_{\mathrm{pr}}(\Delta t,\omega)\right]$$

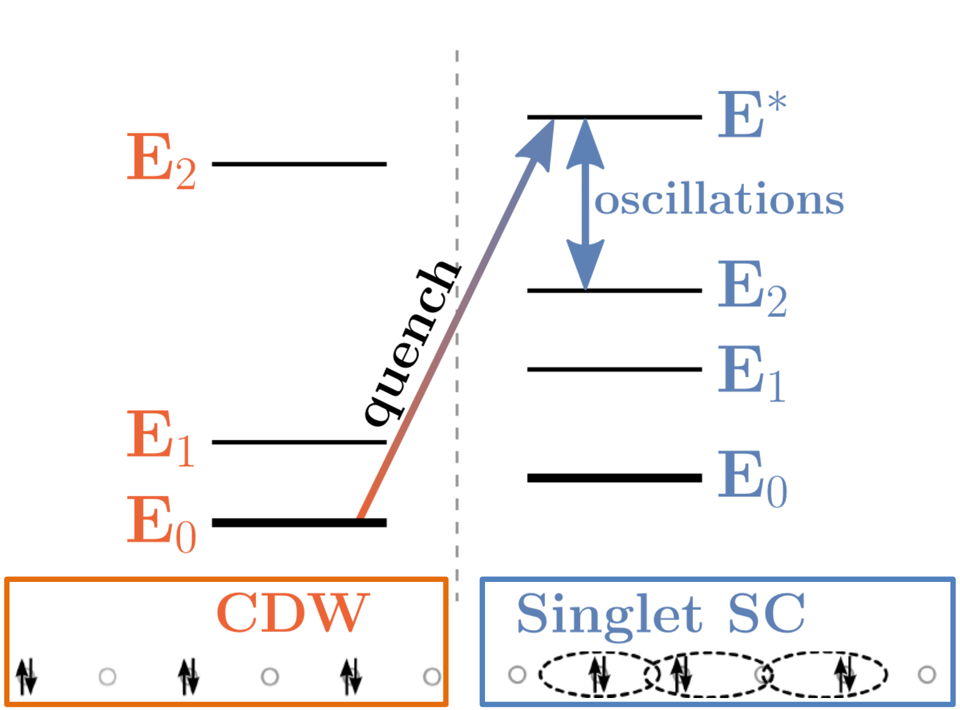
⇒ **Oscillations** in pump-probe response **as a function of delay time Δt**

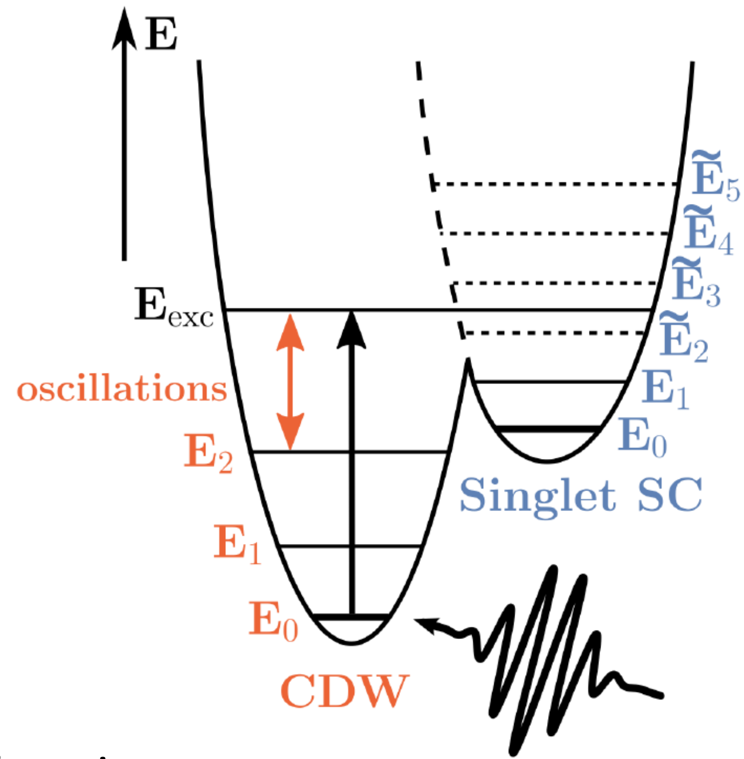




Summary (Part 1): Quench vs. Pulse

Time-dependent solution of the extended Hubbard model:





→ Enhancement of the on-site correlation function

→ Transient Meissner effect

→ Quench: direct into the new phase

→ Pulse: fingerprints of the new phase short after the pulse

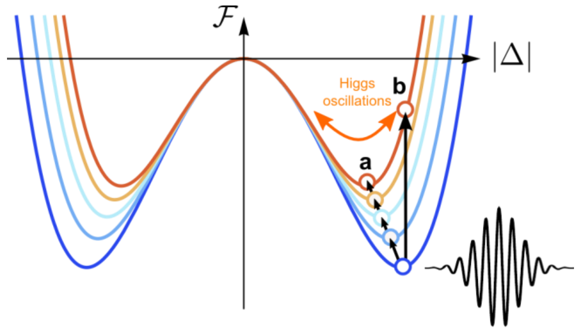
Summary (Part 2)

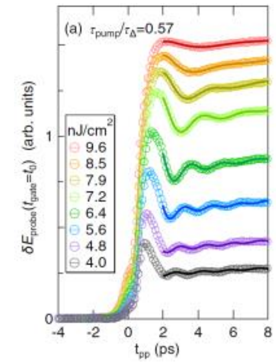
Micoscopic theory for ultrafast dynamics in superconductors employing Density Matrix Theory:

What happens after the laser pulse?

- case 1: single band

» Higgs oscillations in
  the *non-adiabatic* regime, $\tau_p < \tau_\Delta$





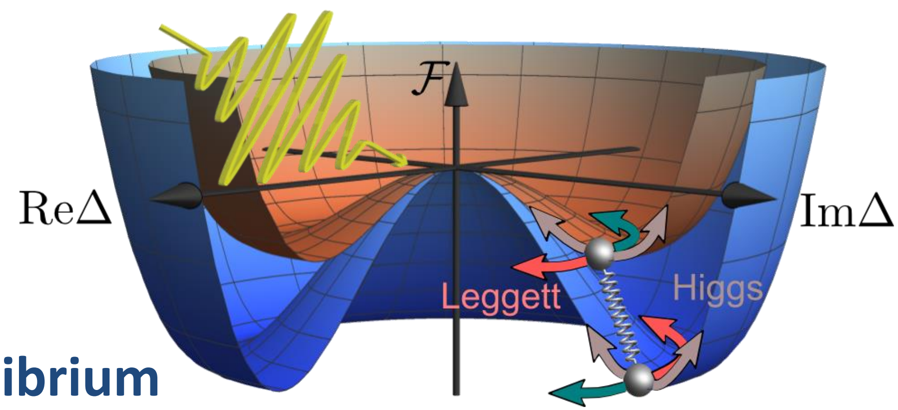
- case 2: two bands (MgB_2): predictions

» Josephson coupling:
  new Leggett mode detected

» new dispersion

» amplitude-phase coupling in nonequilibrium

» higher harmonics can be observed



Thank you for your attention!