Lecture 1: Introduction to exoplanetary transits

Transit probability
Early detections
Transit depths and durations
Model-independent system parameters
Transiting planets

• Struve 1952: Observatory 72, 199.

PROPOSAL FOR A PROJECT OF HIGH-PRECISION STELLAR RADIAL VELOCITY WORK

By Otto Struve

But there seems to be no compelling reason why the hypothetical stellar planets should not, in some instances, be much closer to their parent stars than is the case in the solar system. It would be of interest to test whether there are any such objects.

There would, of course, also be eclipses. Assuming that the mean density of the planet is five times that of the star (which may be optimistic for such a large planet) the projected eclipsed area is about 1/50th of that of the star, and the loss of light in stellar magnitudes is about 0.02. This, too, should be ascertainable by modern photoelectric methods, though the spectrographic test would probably be more accurate. The advantage of the photometric procedure would be its fainter limiting magnitude compared to that of the high-dispersion spectrographic technique.
Transits

Simplest method: look for drop in stellar flux due to a planet transiting across the stellar disc

Venus Transit in 2004

Transits only occur if the orbit is almost edge-on. What is the probability?
Orbital inclination, $i$

Angle OSE, subtended at star $S$

by direction $O$ of orbital pole

and direction $E$ to Earth (along red axis)

$i = 90^\circ \Rightarrow$ equator-on

$i = 0^\circ \Rightarrow$ pole-on
Full and grazing transits

- Grazing transit
  
  \[ a \cos i \leq R_* + R_p \]

- Full transit
  
  \[ a \cos i \leq R_* - R_p \]
Random Orbit Orientation

- Probability that angle between pole of orbit and line of sight lies between $i$ and $i + di$?

$$d(\text{Prob}) = \frac{d\Omega}{4\pi} = \frac{2\pi \sin(i) d(i)}{4\pi} = \frac{d(\cos(i))}{2}$$
Transit surveys find planets in small orbits around large parent stars.

Transits occur only in nearly edge-on orbits:

\[ a \cos i \leq R_* + R_p \]

Random orbit orientation -> probability uniform in \( \cos(i) \).

Transit probability is then:

\[
\text{Prob} \left( \cos i < \frac{R_* + R_p}{a} \right) = \frac{R_* + R_p}{a} \approx \frac{R_*}{a}
\]

Transit surveys find planets in small orbits around large parent stars.
Transit Probability

\[
\text{Prob} \approx \frac{R_\star}{a} \approx 0.005 \left( \frac{R_\star}{R_{\text{sun}}} \right) \left( \frac{1\,\text{AU}}{a} \right)
\]

- Hot planets more likely to be detected.
- Prob = 0.5 % at 1 AU, Prob = 0.1 % at 5 AU (Jupiter’s orbit)
- Prob = 10% at 0.05 AU (Hot Jupiters)
- Thousands of stars must be monitored to discover planets by spotting their transits.
1999 First Transiting Exoplanet

Charbonneau & Brown (2000)

STARE 10 cm telescope

A Very Big Discovery by a grad student using a Very Small Telescope!

HD 209458 V=7.6 mag
1.6% deep “winks”
last 3 hours
repeat every 3.5 days
HD 209458
Transits
HST/STIS

Brown et al. (2001)

\[ P = 3.52 \text{ d} \]
\[ a = 0.046 \text{ AU} \]
\[ m_V = 7.8 \]
\[ \Delta f / f = 0.017 \text{ mag (1.6\%)} \]
\[ i = 86.6 \pm 0.2^\circ \]
\[ r_p = 1.35 \pm 0.06 \text{ r}_J \]

From radial velocities
\[ m \sin i = 0.69 \text{ m}_J \]
⇒ “bloated” gas giant

Vietri Sul Mare 2015
Transit Depth

What fraction of the star’s disk does the planet cover?

$$\Delta f \approx \left( \frac{r_p}{R_*} \right)^2 = 0.01 \left( \frac{r_p}{r_{Jup}} \right)^2 \left( \frac{R_*}{R_{sun}} \right)^{-2}$$

Find star radius from its spectral type.

Observed depth yields planet/star area ratio.
Transit duration

- Winn 2008, IAU Symp. 253
Transit duration and stellar density

- Simplest case: circular orbit, $i=90$ degrees
  - Relative transit duration:
    \[
    \frac{T}{P} \approx \frac{2R_*}{2\pi a}
    \]
  - Kepler’s 3rd Law:
    \[
    a^3 = GM_* \left( \frac{P}{2\pi} \right)^2
    \]
    Hence
    \[
    \frac{T}{P} \approx \frac{R_*}{\pi a} = \frac{R_*}{\pi} \left( \frac{4\pi^2}{GM_*P^2} \right)^{1/3}
    \]
    \[
    T \approx 3h \left( \frac{P}{4d} \right)^{1/3} \left( \frac{\rho_*}{\rho_{\text{Sun}}} \right)^{-1/3}
    \]
Planetary surface gravity

- Simplest case: circular orbit, $i=90$ degrees
  - Stellar orbital acceleration:
    \[
    \frac{dv}{dt} = \frac{2\pi K}{P} = \frac{GM_p}{a^2} = g_p \frac{R_p^2}{a^2}
    \]
  - Inverse square law of gravitation:
    \[
    \frac{2\pi K}{P} = g_p \left( \frac{R_p}{R_*} \right)^2 \left( \frac{R_*}{a} \right)^2
    \]

Vietri Sul Mare 2015
Planetary density

- Need to know planet radius and surface gravity:
  \[ g_p = \frac{GM_p}{R_p^2} = GR_p\rho_p = GR_\ast\left(\frac{R_p}{R_\ast}\right)\rho_p \]

- Use stellar angular diameter \( \theta \) and parallax \( \pi \):
  \[ R_\ast = \theta d = \frac{\theta}{\pi} \]

- Hence get planetary bulk density:
  \[ \rho_p = \frac{g_p\pi}{G\theta}\left(\frac{R_\ast}{R_p}\right) \]
Transit Duration ($i < 90^\circ$)

Transit duration reduces to 0 as orbit tips away from edge-on.

Time from first to last contact:

$$t_T = \frac{P}{\pi} \arcsin \left( \frac{R_*}{a} \sqrt{\left( 1 + \frac{R_p}{R_*} \right)^2 - \left( \frac{a}{R_*} \cos i \right)^2} \right)$$

For $\cos i \ll 1$ this becomes:

$$t_T = \frac{P R_*}{\pi a} \sqrt{\left( 1 + \frac{R_p}{R_*} \right)^2 - \left( \frac{a}{R_*} \cos i \right)^2}$$

Transit Duration

\[ t_T = \frac{P R_*}{\pi a} \sqrt{\left(1 + \frac{R_p}{R_*}\right)^2 - \left(\frac{a}{R_*} \cos i\right)^2}. \]

Inclination determines impact parameter, \( b = a \cos i / R \) hence shape of lightcurve.

Fundamental derived quantities

- Winn 2008, IAU Symp. 253

Radius ratio \( R_p/R_s \approx \sqrt{\delta} \),

Impact parameter \( b \approx 1 - \sqrt{\delta} \frac{T}{\tau} \),

Scaled stellar radius \( R_s/a \approx \frac{\pi \sqrt{T\tau}}{\delta^{1/4} P} \left( \frac{1 + e \sin \omega}{\sqrt{1 - e^2}} \right) \)
Model-independent parameters

- Winn 2008, IAU Symp. 253

Radius ratio \( R_p/R_s \approx \sqrt{\delta} \)

Impact parameter \( b \approx 1 - \sqrt{\delta} \frac{T}{\tau} \)

Scaled stellar radius \( R_s/a \approx \frac{\pi\sqrt{T\tau}}{\delta^{1/4} P} \left( \frac{1 + e \sin \omega}{\sqrt{1 - e^2}} \right) \)

Stellar mean density \( \rho_s \approx \frac{3P}{\pi^2 G} \left( \frac{\sqrt{\delta}}{T\tau} \right)^{3/2} \left[ \frac{1 - e^2}{(1 + e \sin \omega)^2} \right]^{3/2} \)

Planetary surface gravity \( g_p \approx \frac{2\pi K_s}{P} \frac{\sqrt{1 - e^2}}{\delta (R_s/a)^2 \sin i} \)
(1) Spectral Type gives star mass and radius.

(2) Period (+ Kepler’s law) gives orbit size.

(3) Depth of transit gives planet radius.

Models of planets with masses between ~ 0.1 M$_J$ and 10 M$_J$, have almost the same radii (i.e. a flat mass-radius relation).

-> Giant planets transiting solar-type stars expected to have transits depths of around 1%

(4) Impact parameter $b = a \cos(i)/R$, determined from the shape of the transit, gives a measure of inclination angle.

(5) Bottom of light curve is not flat in all wave bands, providing a measure of stellar limb-darkening

(6) Since inclination is measured, can measure mass, not just lower limit $m_p \sin(i)$, from the radial velocity data.
Summary

• Transit probability is \( \sim R_/\cdot /a \)
  – 10% for hot Jupiter, 0.5% for Earth transiting Sun

• Transit depth is \( \sim (R_/p_/R_/\cdot)^2 \)
  – 1% for Jupiter transiting Sun

• Transit duration is \( \sim R_/\cdot /a \)
  – Gives stellar density via Kepler 3

• Transit depth, duration and radial acceleration of star yield planetary surface gravity.

• To get planet density, need to know stellar radius
  – Use stellar models,
  – OR stellar angular diameter + parallax (Gaia)

• Impact parameter \( b = a \cos(i)/R_/\cdot \)

• Eccentricity modifies transit duration
  – Need to know orbital parameters