The Radial Velocity (RV) Method for the Detection of Exoplanets

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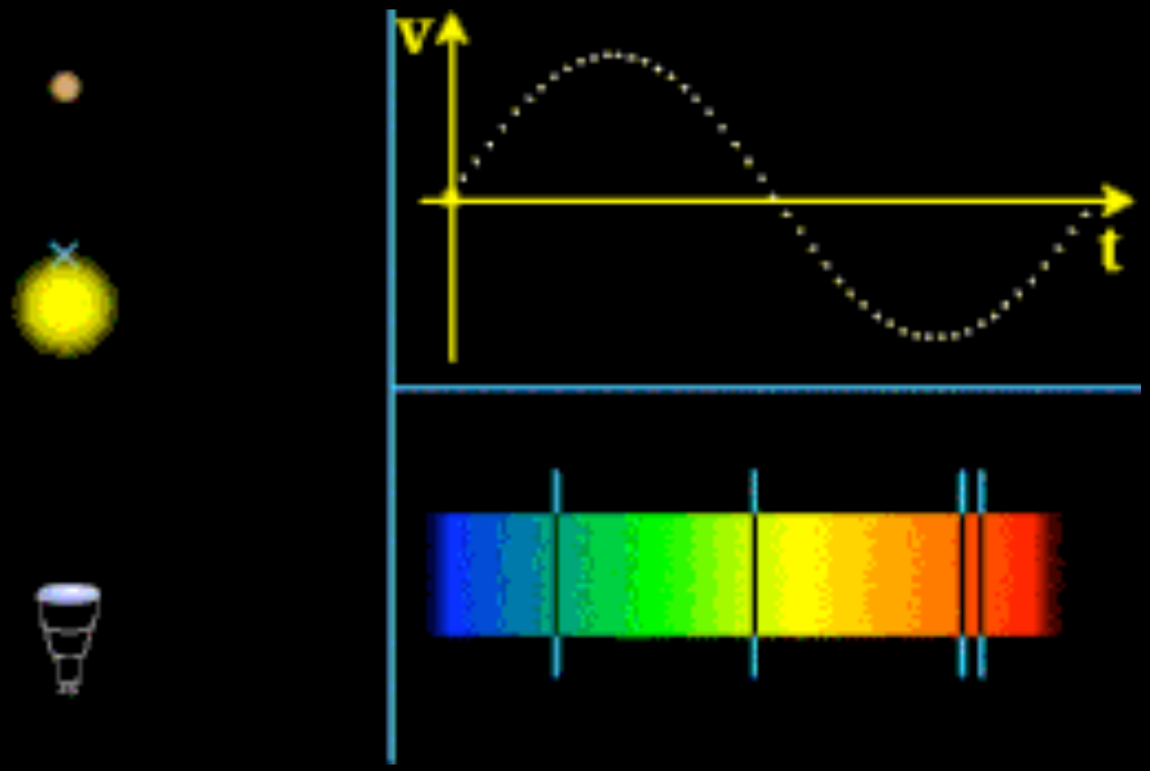
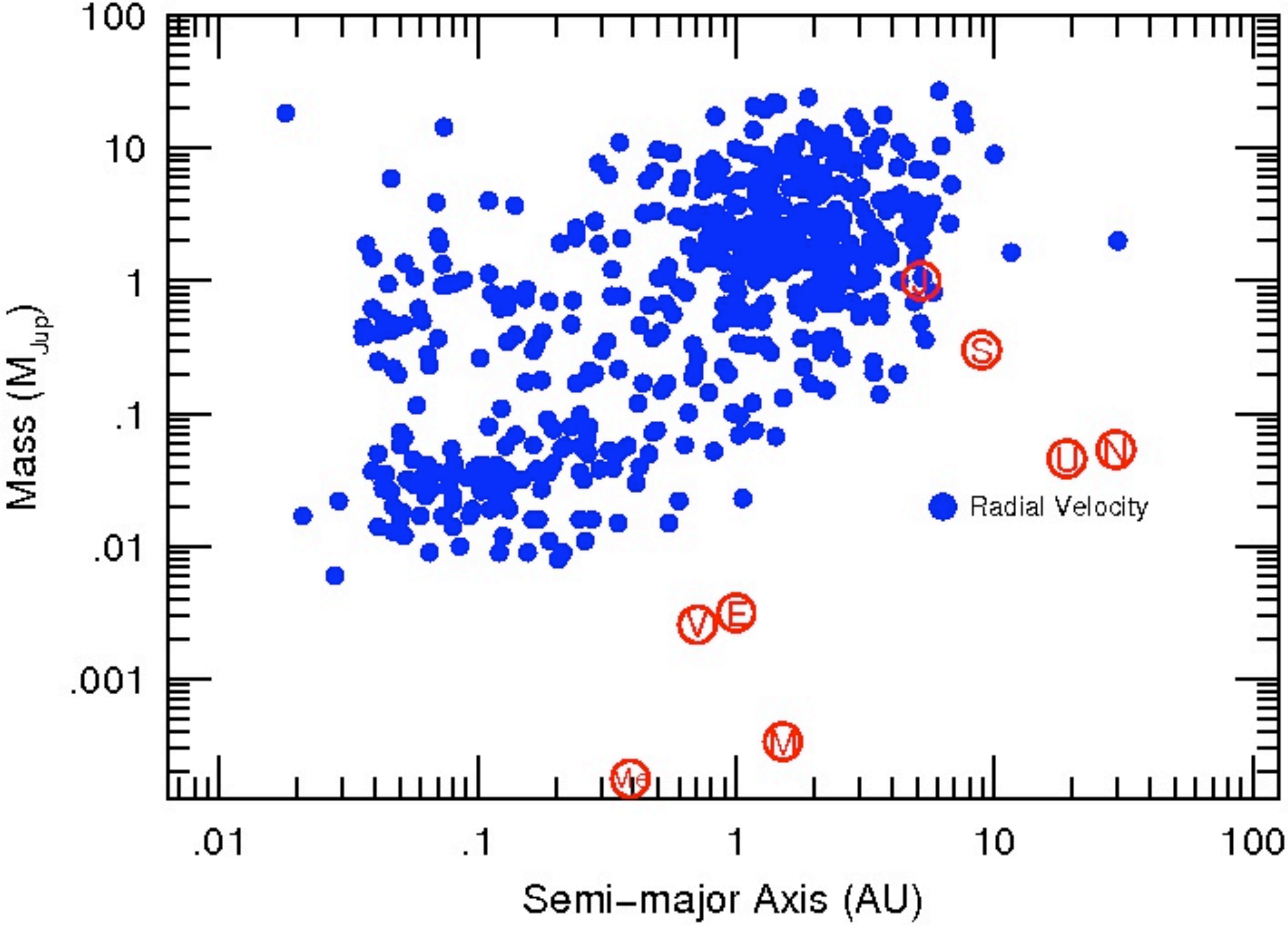


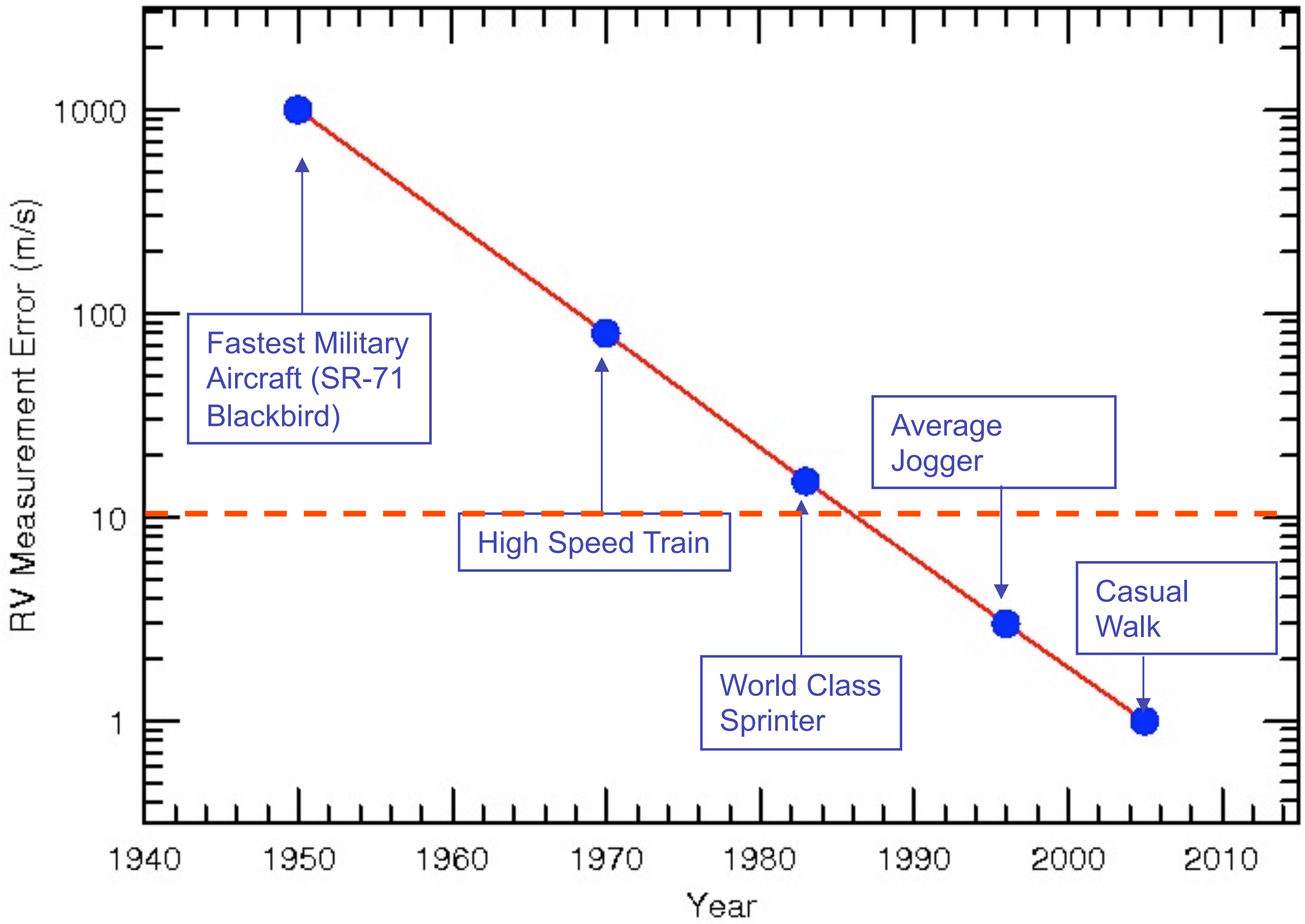
Outline of RV Lectures

1. Instrumentation: Detectors and Spectrographs
2. Precise Stellar Radial Velocity Measurements
3. Frequency Analysis: Detecting Periodic Signals in Time Series Data
4. Keplerian Orbits
5. Sources of Errors: Instrumental and Stellar
6. Dealing with the Activity Signal

Exoplanet Discovery Space



The Radial Velocity Measurement Error with Time

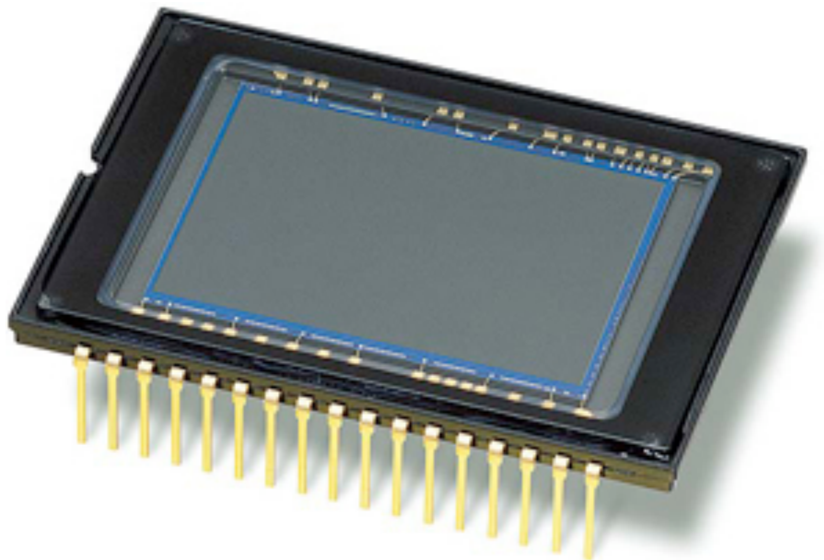


RV precision of 10 cm/s by 2023!

This was achieved by a combination of:

1. **CCD Detectors**
2. **Cross-dispersed echelle spectrographs**
3. Better wavelength calibration
4. Reduction of instrumental shifts

Charge Coupled Devices



• Quantum Efficiency of 80-90% (Photographic plates ~1%) ➔ Higher S/N data

• 2-D Arrays ➔ Perfect match to echelle spectrographs

• Data in digital form ➔ easier manipulation of data

The Basic Unit of a CCD: The Pixel

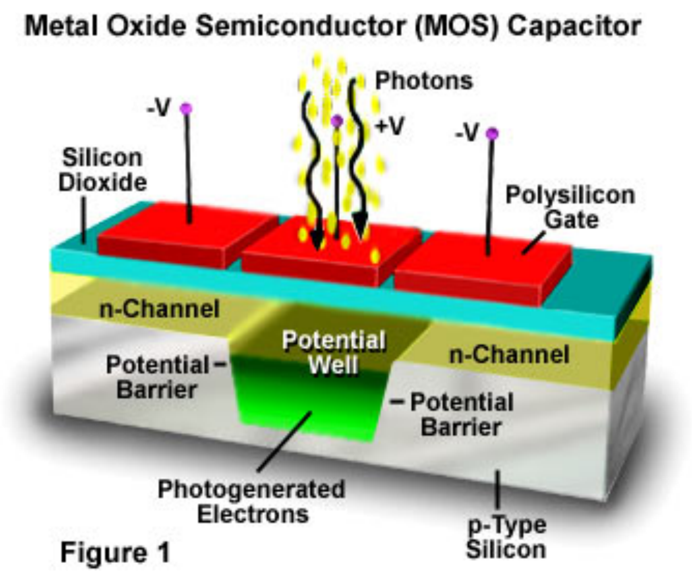


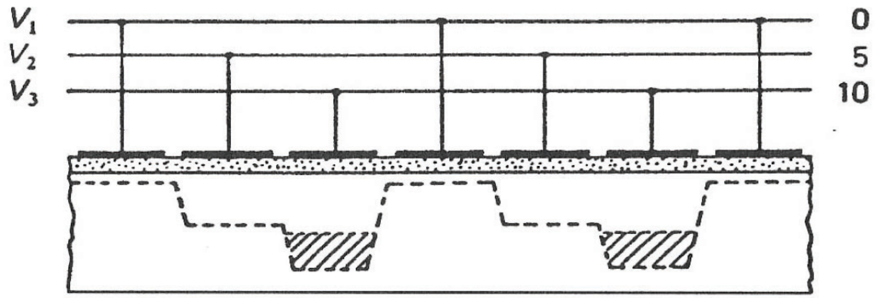
Figure 1

The basic element of a CCD consists of a Metal Oxide Semiconductor. The bulk material is p-silicon on which an insulating layer of silicon oxide has been grown as well as thin conducting electrodes of transparent polysilicon. The central electrode is set to a positive bias while the two flanking electrodes are negative. This creates a "depletion" region under the central electrode containing no holes but a deep potential well to trap electrons. The region shown is about 10μ thick. During exposure light enters through the "front-side" electrodes. Photoelectrons generated under the central electrode will be attracted toward the electrode and held below it. The corresponding holes will be swept away into the bulk silicon.

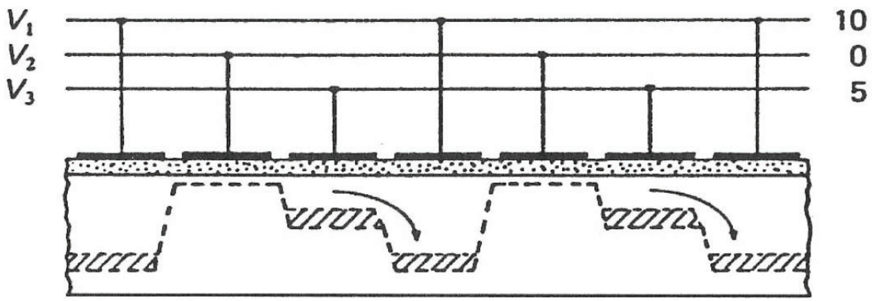
From http://www.astro.virginia.edu/class/oconnell/astr511/lec11-f03.html

Reading out a CCD

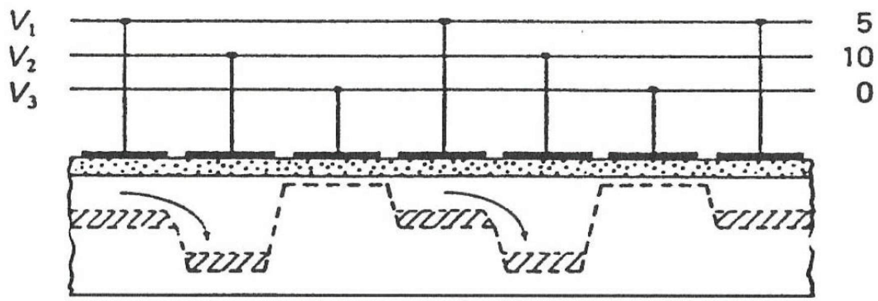
A „3-phase CCD“



End of exposure



Charge transfer



Columns →

Parallel registers shift the charge along columns

There is one serial register at the end which reads the charge along the final row and records it to a computer

For last row, shift is done along the row

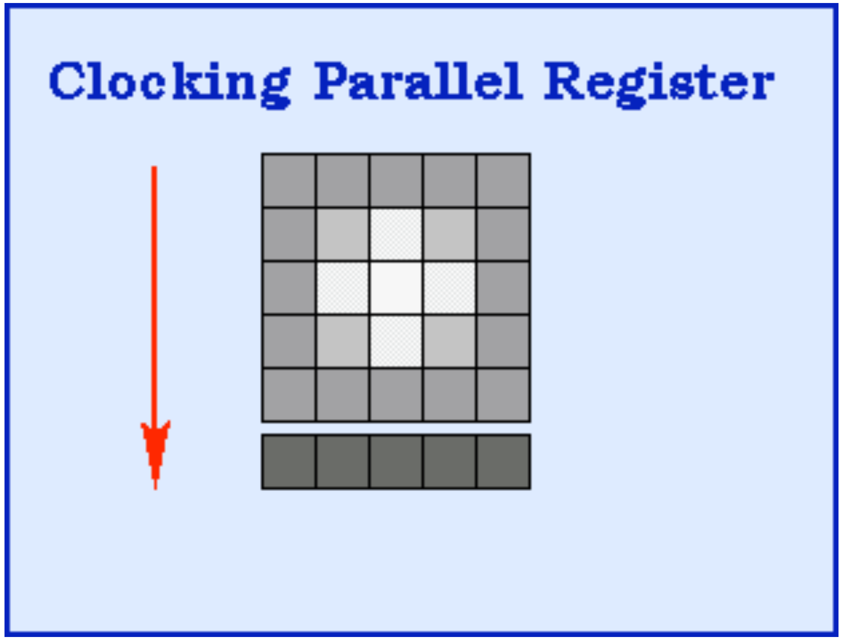


Figure from O'Connell's lecture notes on detectors

The CCD is first clocked along the parallel register to shift the charge down a column

The CCD is then clocked along a serial register to readout the last row of the CCD

The process continues until the CCD is fully read out.

How much charge is lost in the this charge transfer process?

Typical Charge Transfer Efficiency of a CCD is >99.999 %

Suppose you have a 4096 x 4096 CCD and detect 40.000 photons (electrons). Signal to Noise ratio = $\sqrt{N}$ = 200

Charge recorded = $40.000 \times 0.99999^{4096} = 38.394$

1605 electrons „lost“

S/N decreased to 195

CCD Parameters Important for Observations

- Gain: Converts ADU to number of photons detected. Important for Signal-to-Noise estimate. Typically 0.5–10 e^{-1}/ADU
- Linearity: Detected counts should be proportional to the exposure time. If a CCD has a non-linear regime these level of counts should be avoided
- Readout Noise: Noise introduced by CCD readout electronics. Negligible for High Signal-to Noise observations
- Dark: Thermal noise. Neglible for High Signal-to-Noise Observations Most science grades CCDs are kept at –120 C or cooler.
- Bias level: Constant level added to the data by the electronics to ensure that there are no negative numbers

McDonald CCD for coude spectrograph:

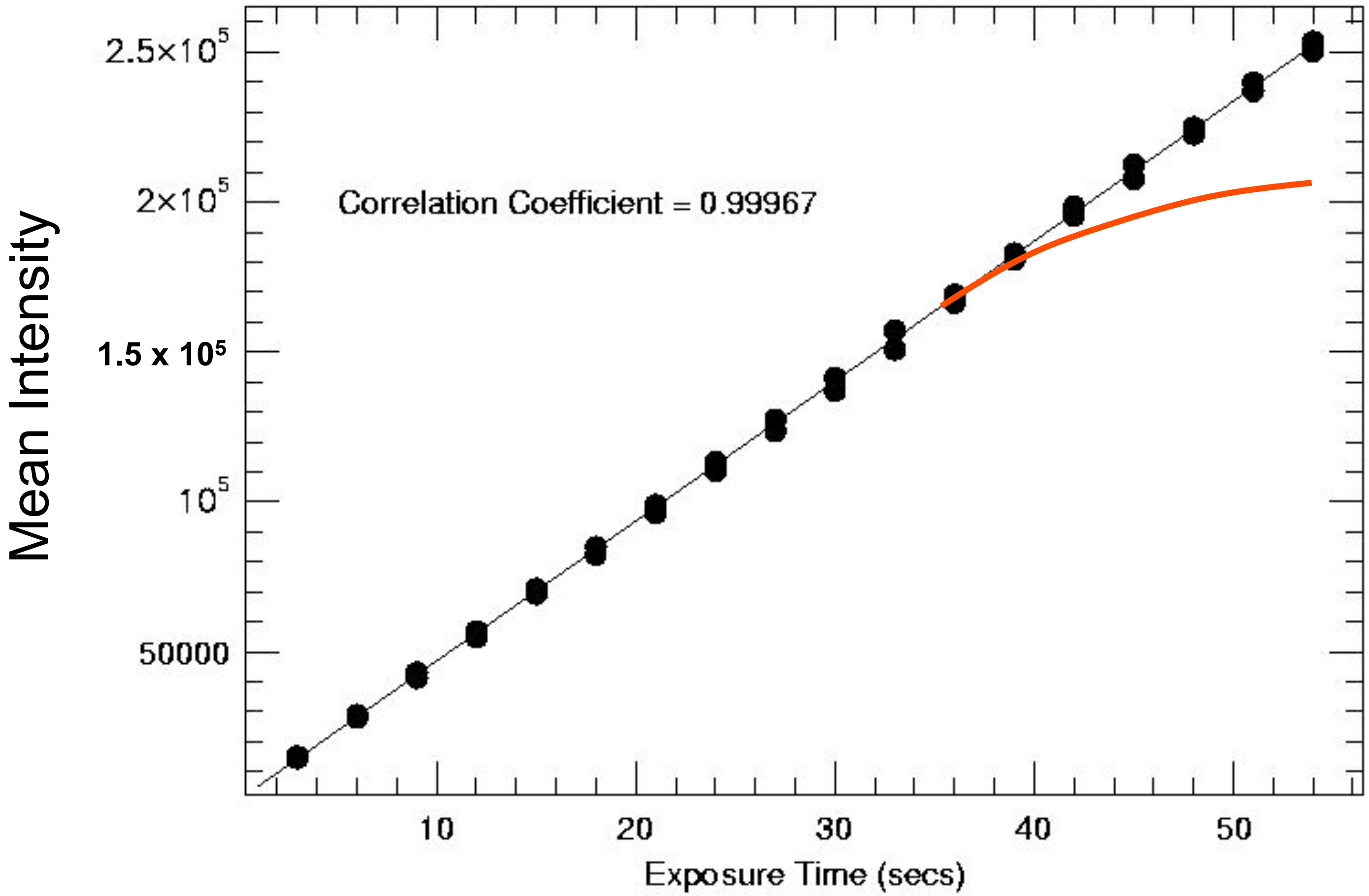
Gain = 0.56 ± 0.015 e^{-1}/ADU

Readout Noise = 3.06 electrons

Bias level = 1024

Noise Tests for CCD: Linearity

Take a series of frames of a low intensity lamp and plot the mean counts as a function of exposure time

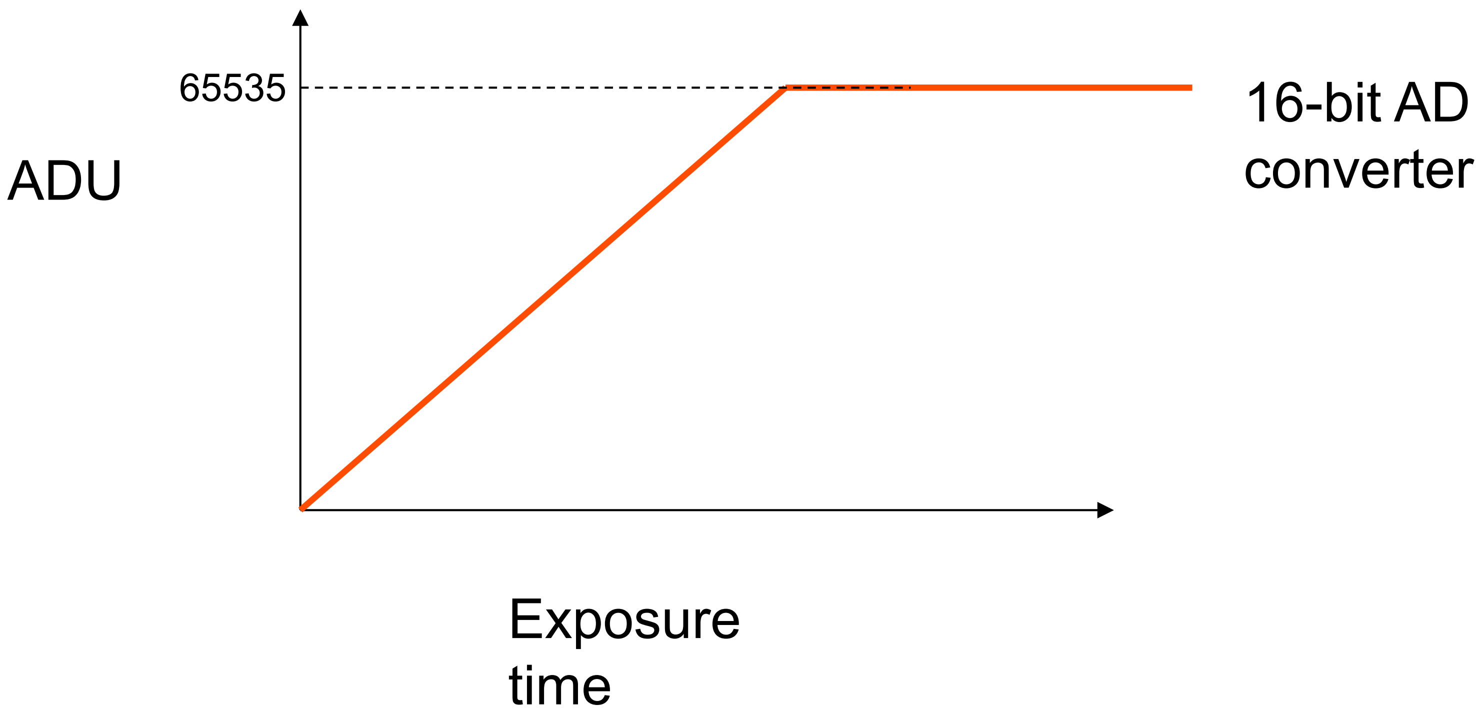


If the curve followed the red line at the high count rate end (and some CCDs do!) then you would know to keep your exposure to under 150.000

Problems and Pitfalls of CCD Usage

Saturation

If too many electrons are produced (too high intensity level) then the full well of the CCD is reached and the maximum count level will be obtained. Additional detected photons will not increase the measured intensity level:

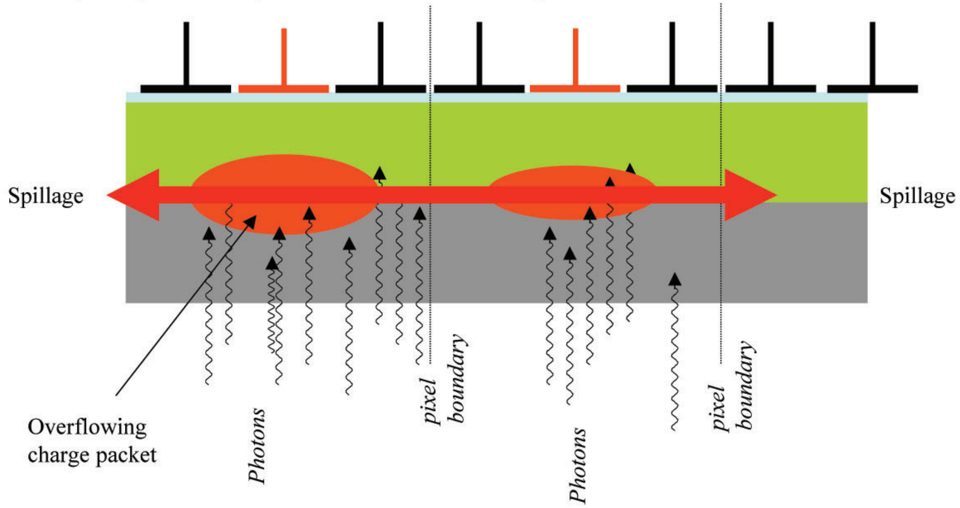


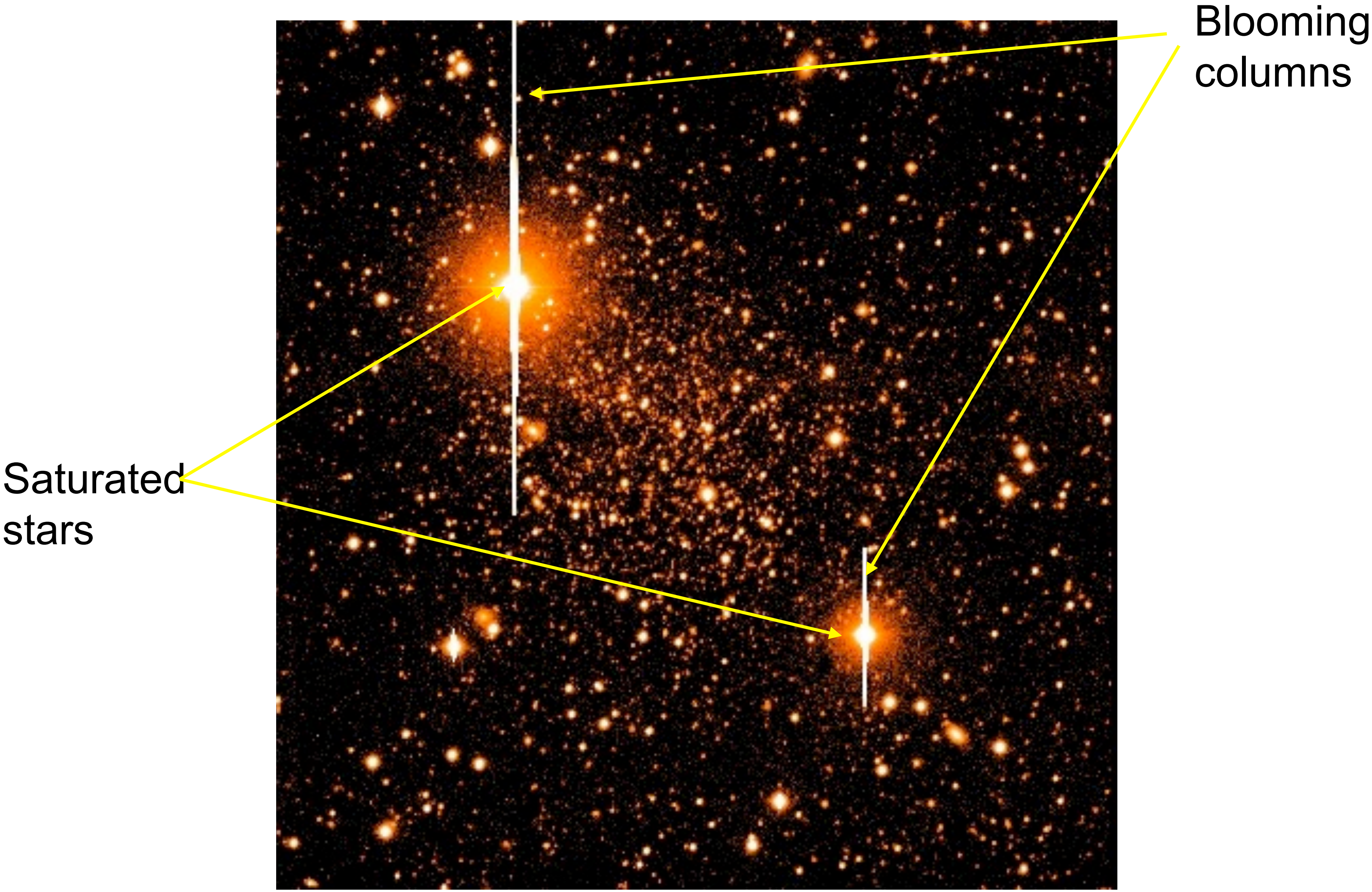
Problems and Pitfalls of CCDs

Blooming:

If the full well is exceeded then charge starts to spill over in the readout direction, i.e. columns. This can destroy data far away from the saturated pixels.

The charge capacity of a CCD pixel is limited, when a pixel is full the charge starts to leak into adjacent pixels. This process is known as 'Blooming'.




Blooming columns
Saturated stars

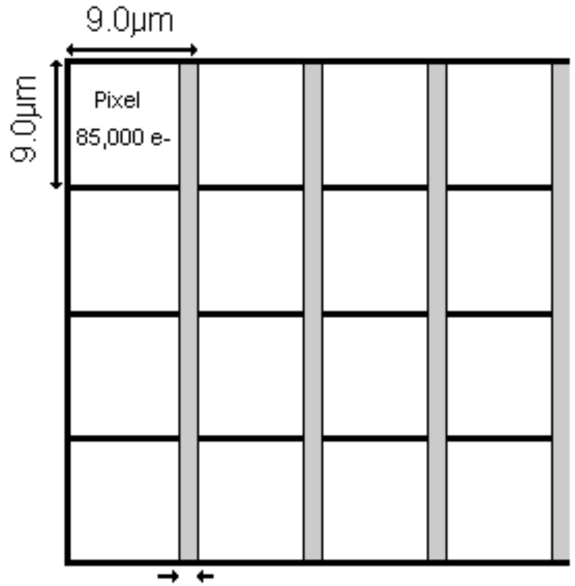
Anti-blooming CCD can eliminate this effect:

Blooming

No blooming

One solution: Anti-blooming CCDs

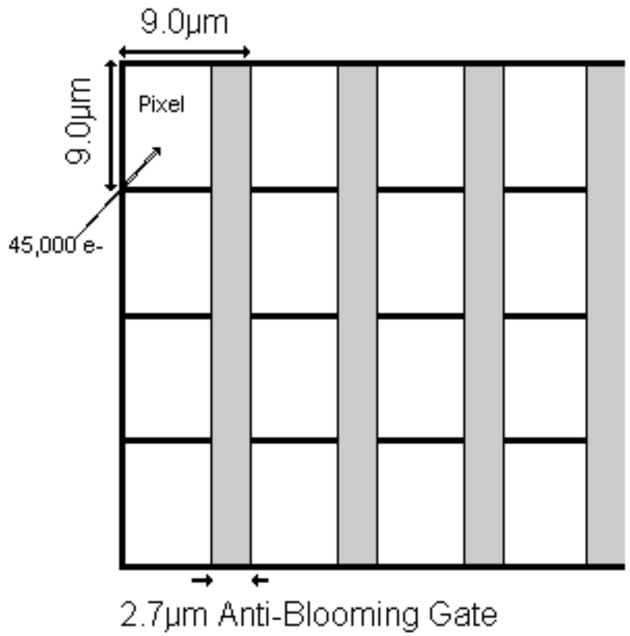
Anti-blooming CCDs have additional gates to bleed off the overflow due to saturation



No Anti-Blooming Gate

100% Fill Factor
85,000 electron well depth
Higher Quantum Efficiency
Blooming (Streaking) possible

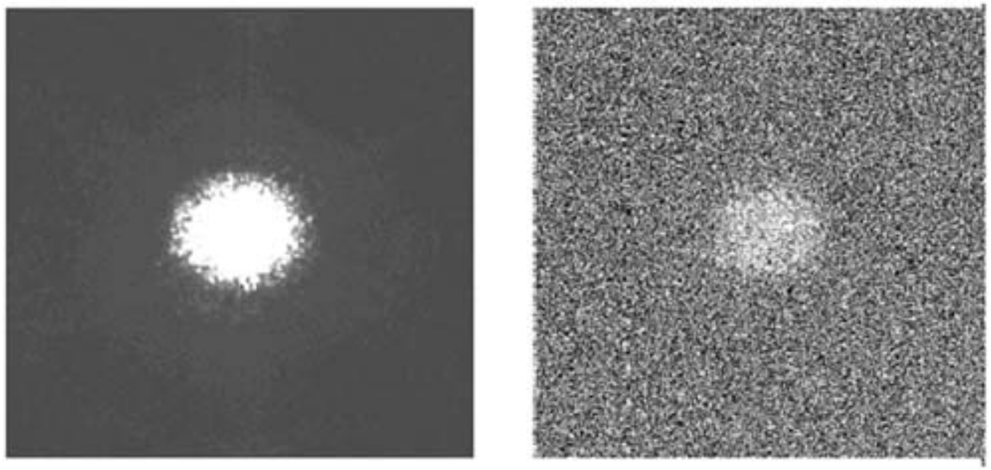
The problem is these gates cover 30% of the pixel. This results in reduced sensitivity, smaller well depth, and lower resolution (gaps between pixels has increased)



Anti-Blooming Gate

70% Fill Factor
45,000 electron well depth
Lower Quantum Efficiency

Residual Images

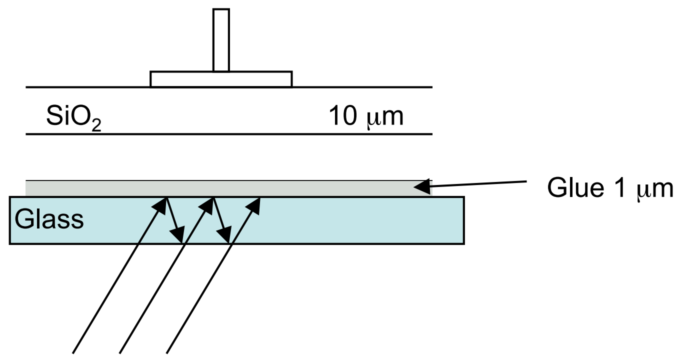


If the intensity is too high this will leave a residual image. Left is a normal CCD image. Right is a bias frame showing residual charge in the CCD. This can effect photometry and spectroscopy.

Solution: several dark frames readout or shift image between successive exposures

Fringing

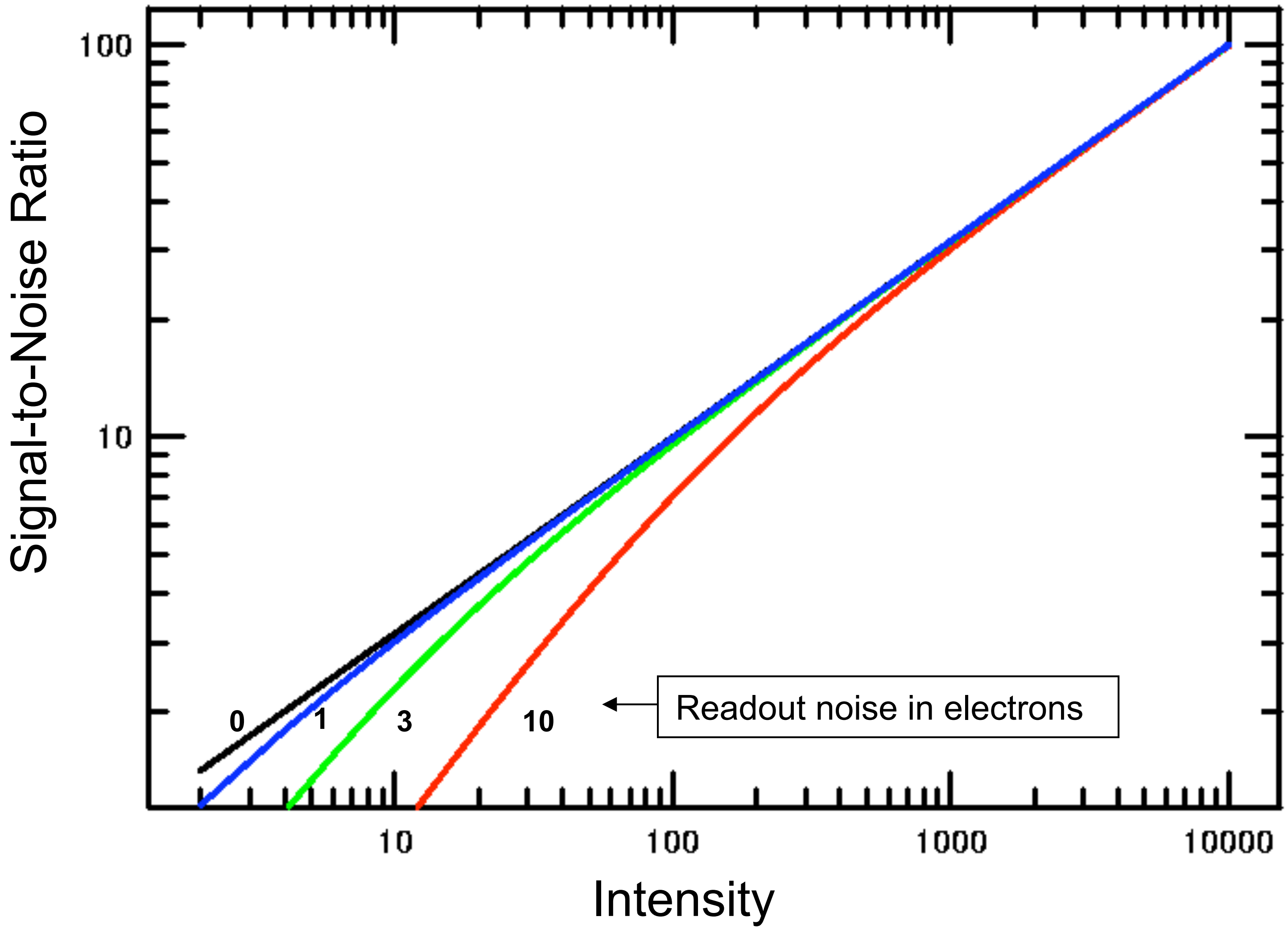
CCDs especially back illuminated ones are bonded to a glass plate



When the glass is illuminated by monochromatic light it creates a fringe pattern. Fringing can also occur without a glass plate due to the thickness of the CCD

Depending on the CCD fringing becomes important for wavelengths greater than about 6500 Å

Readout Noise



High readout noise CCDs (older ones) could seriously affect the Signal-to-Noise ratios of observations

Basic CCD reductions

• Subtract the Bias level. The bias level is an artificial constant added in the electronics to ensure that there are no negative pixels

• Divide by a Flat lamp to ensure that there are no pixel to pixel variations

• Optional: Removal of cosmic rays. These are high energy particles from space that create „hot pixels" on your detector. Also can be caused by natural radioactive decay on the earth.

For Spectral Observations:

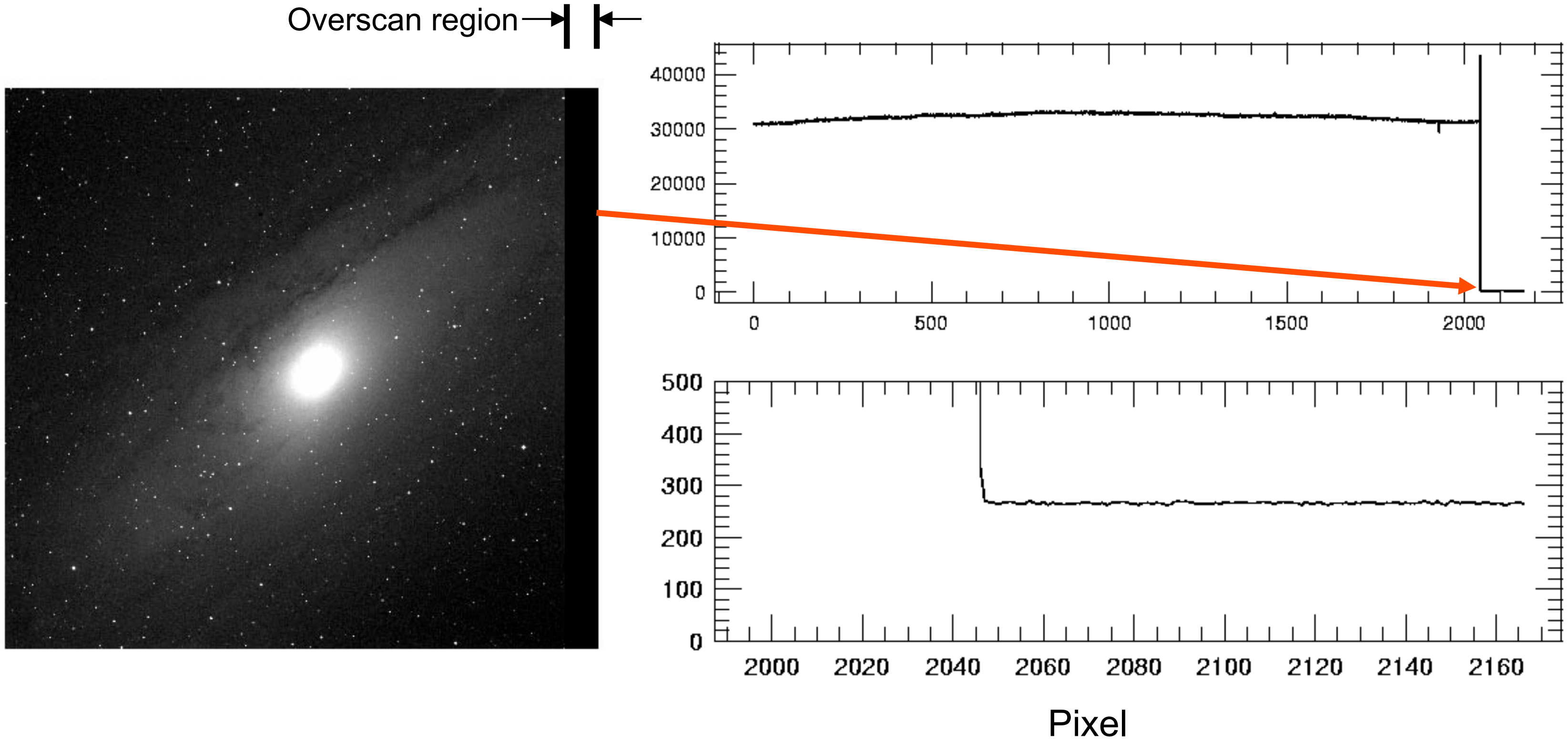
• Global scattered light subtraction

• Order tracing and fitting

• Order extraction

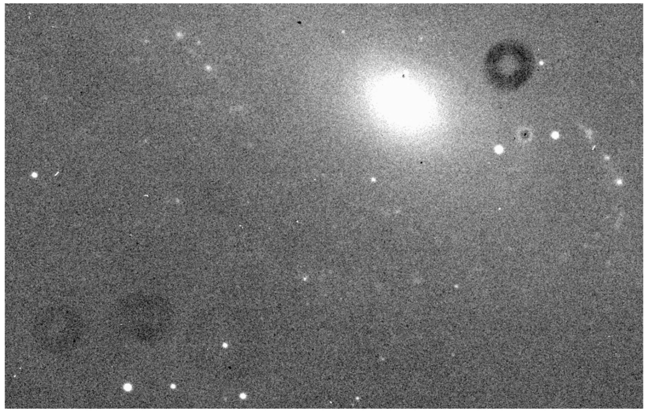
• Wavelength calibration using (typically) a Th-Ar hollow cathode lamp

Bias

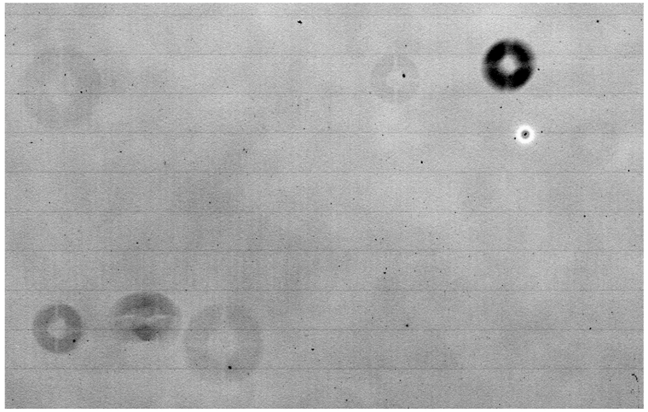


Most CCDs have an overscan region, a portion of the chip that is not exposed so as to record the bias level. The prefered way is to record a separate bias (a dark with 0 sec exposure) frame and fit a surface to this. This is then subtracted from every frame as the first step in the reduction. If the bias changes with time then it is better to use the overscan region

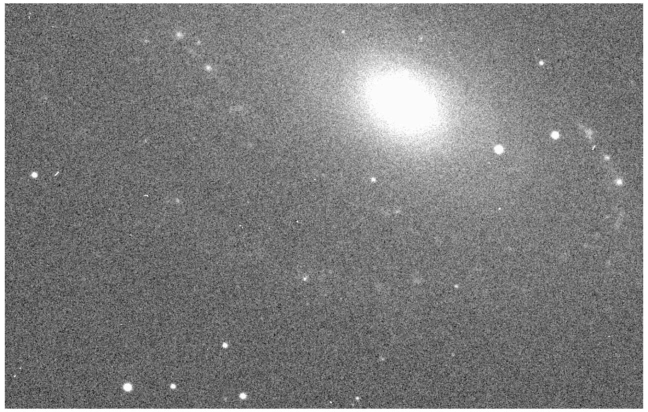
Flat Field Division



Raw Frame

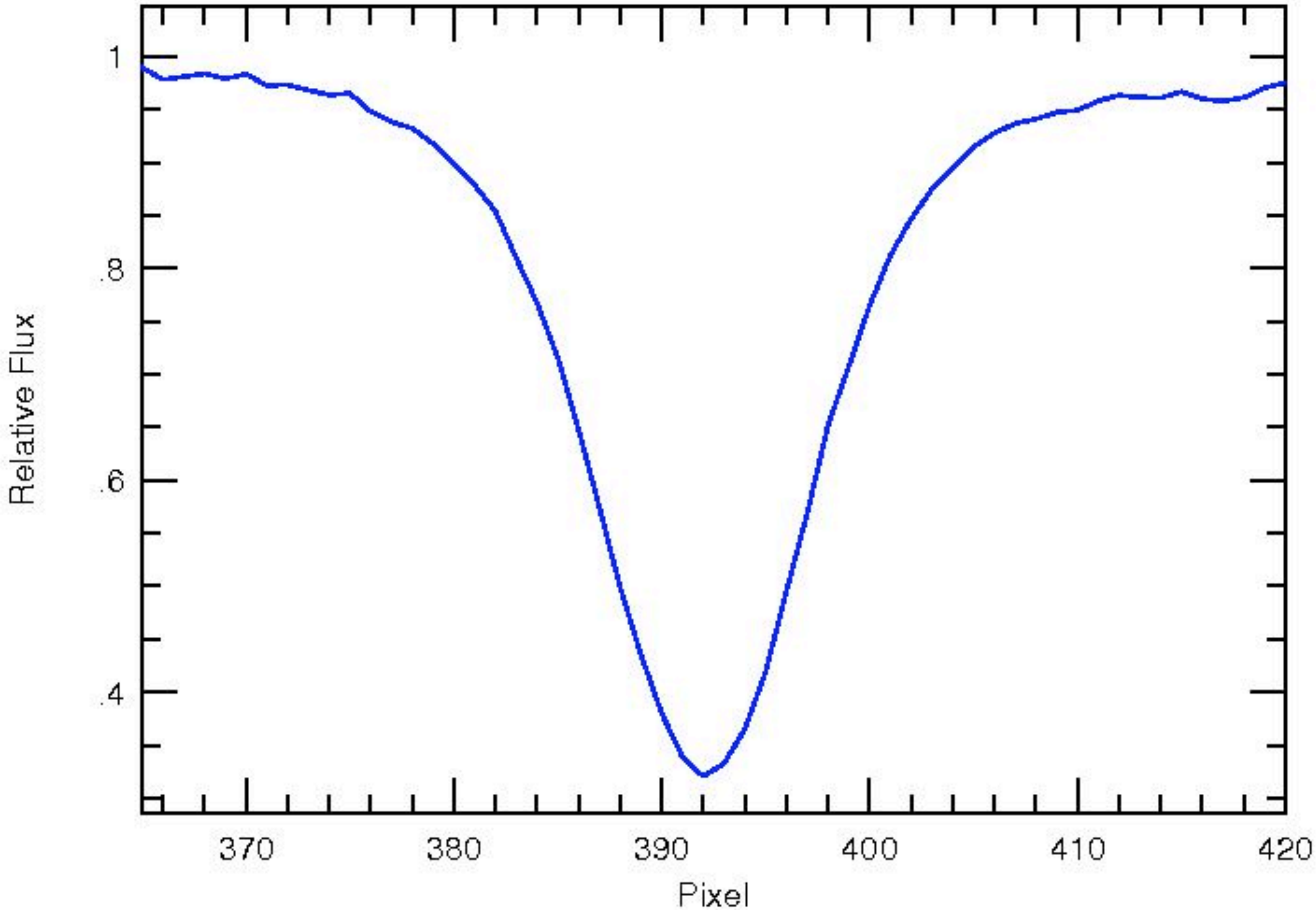


Flat Field

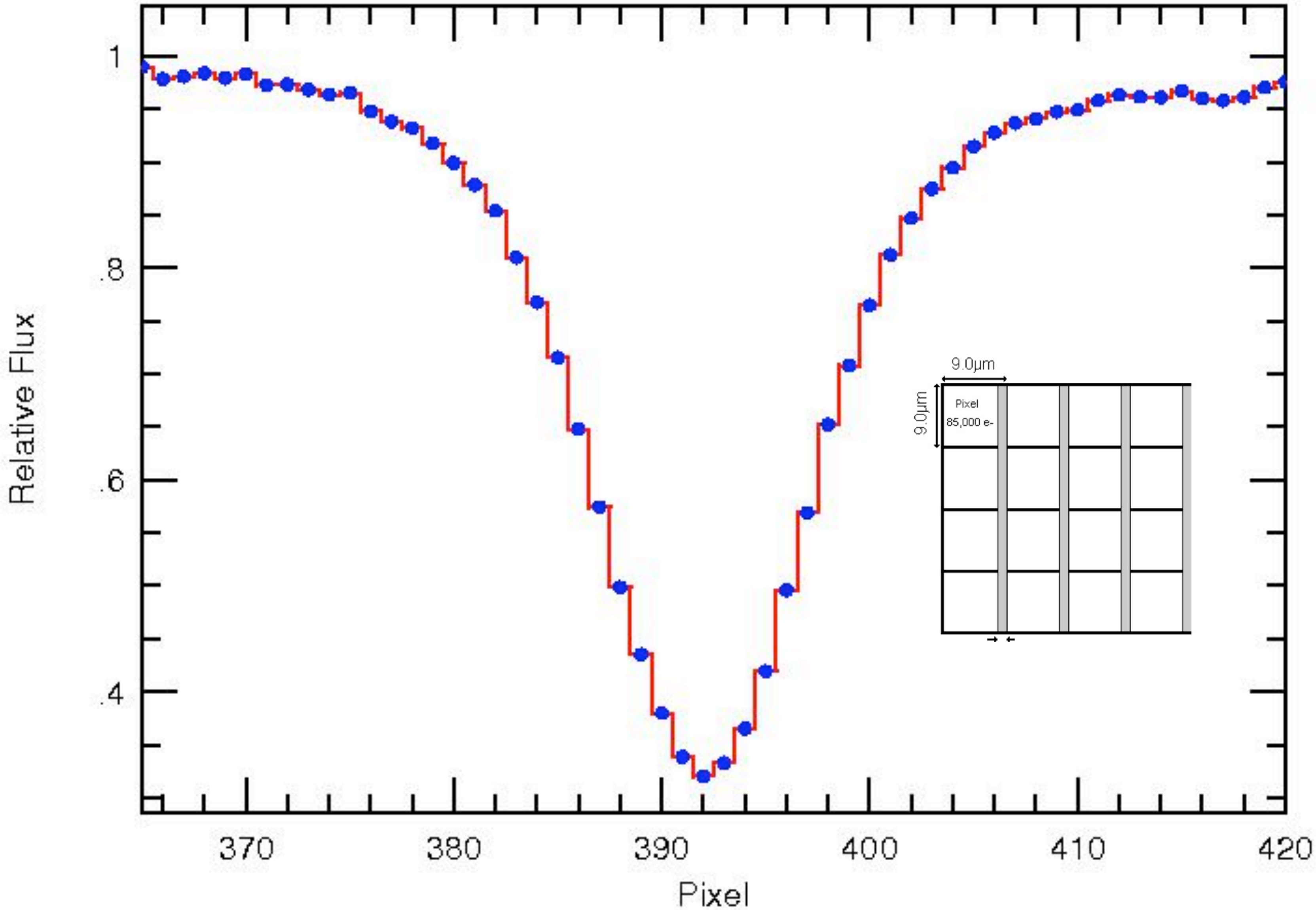


Raw divided by Flat

Every CCD has different pixel-to-pixel sensitivity, defects, dust particles, etc that not only make the image look bad, but if the sensitivity of pixels change with time can influence your results. *Every* observation must be divided by a flat field after bias subtraction. The flat field is an observation of a white lamp. For imaging one must take either sky flats, or dome flats (an illuminated white screen or dome observed with the telescope). For spectral observations „internal" lamps (i.e. ones that illuminate the spectrograph, but not observed through the telescope are taken. Often even for spectroscopy „dome flats" produce better results, particularly if you want to minimize fringing.



1
.8
.6
.4
Relative Flux
370
380
390
400
410
420
Pixel



1
.8
.6
.4
Relative Flux
370
380
390
400
410
420
Pixel
9.0µm
9.0µm
Pixel
85,000 e-

Spectrographs

Spectrographs

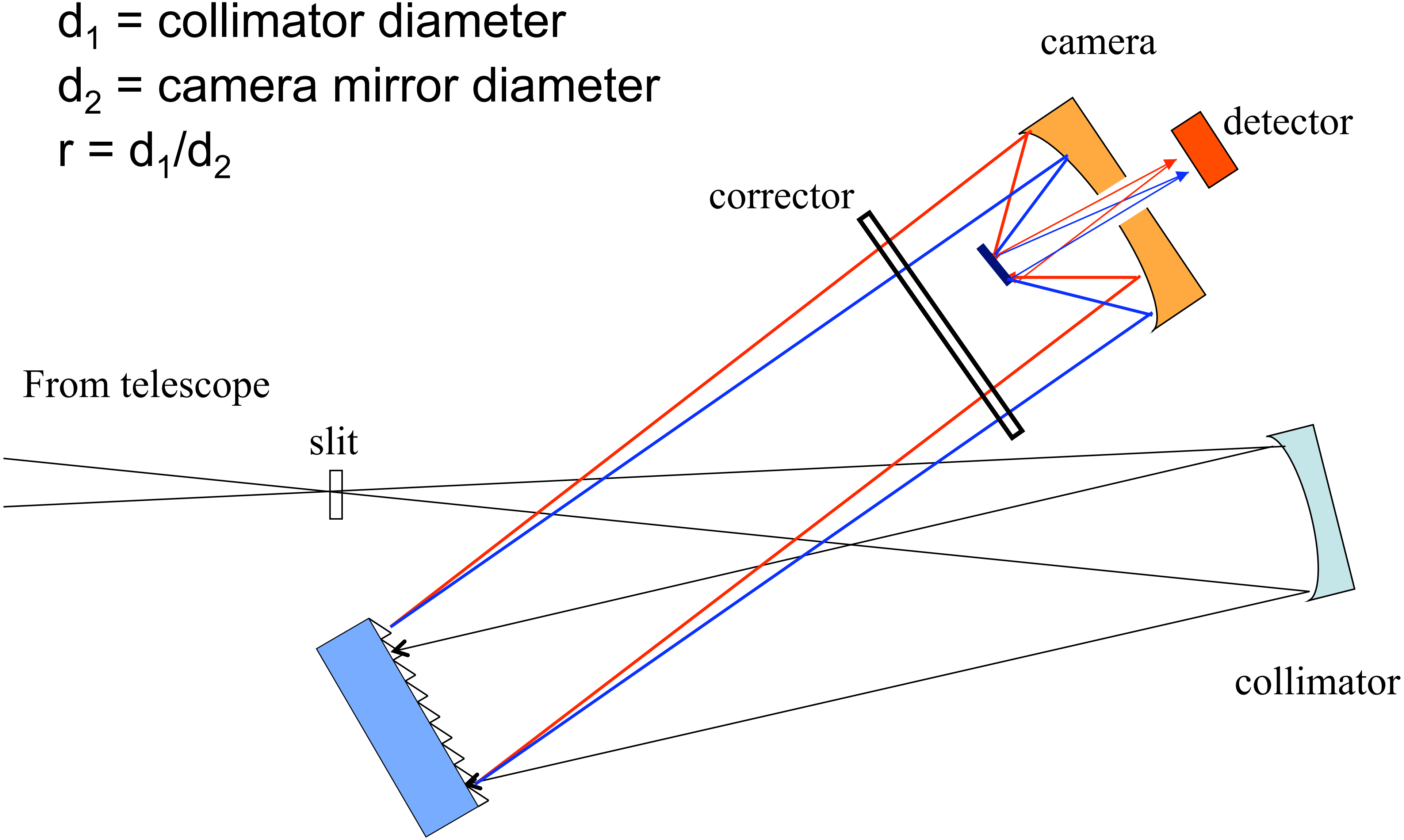
Camera can either be a mirror or lens camera

Anamorphic magnification:

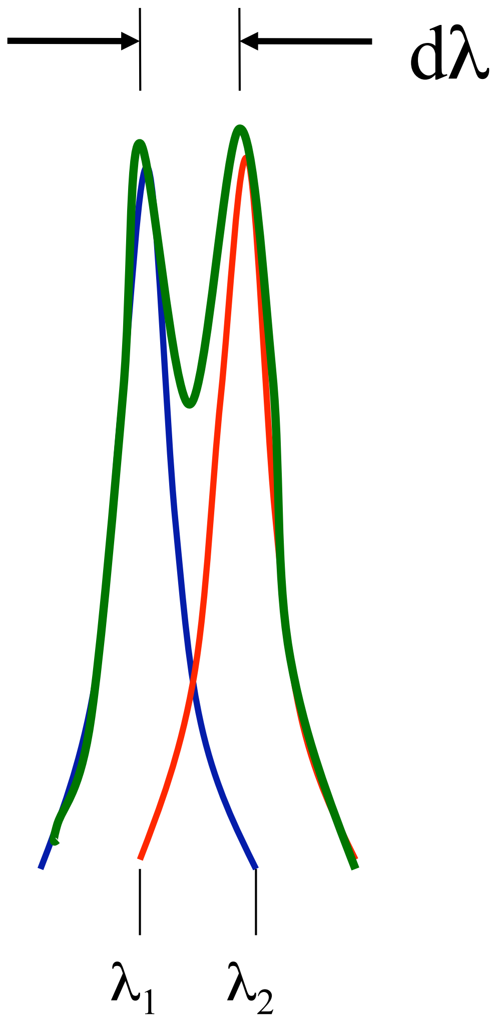
d_1 = collimator diameter

d_2 = camera mirror diameter

$r = d_1/d_2$



Spectral Resolution



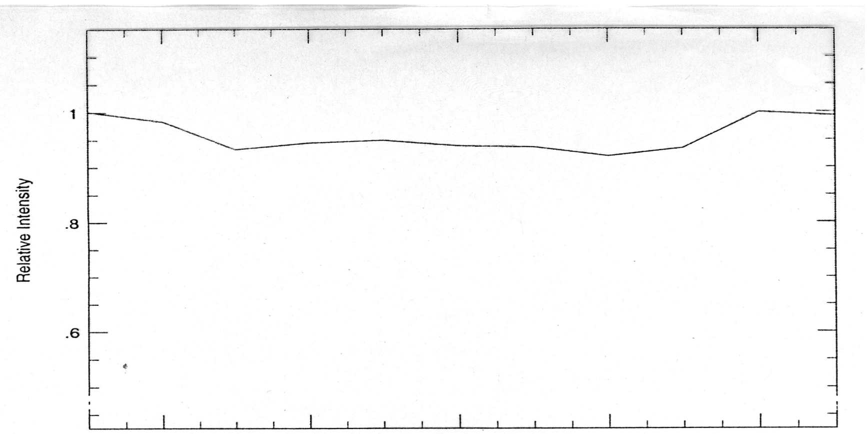
Consider two monochromatic beams

They will just be resolved when they have a wavelength separation of $d\lambda$

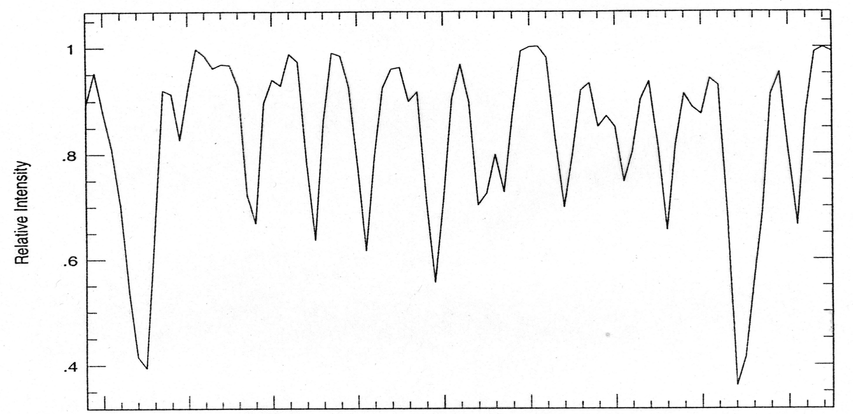
Resolving power:

$$\boxed{R = \frac{\lambda}{d\lambda}}$$

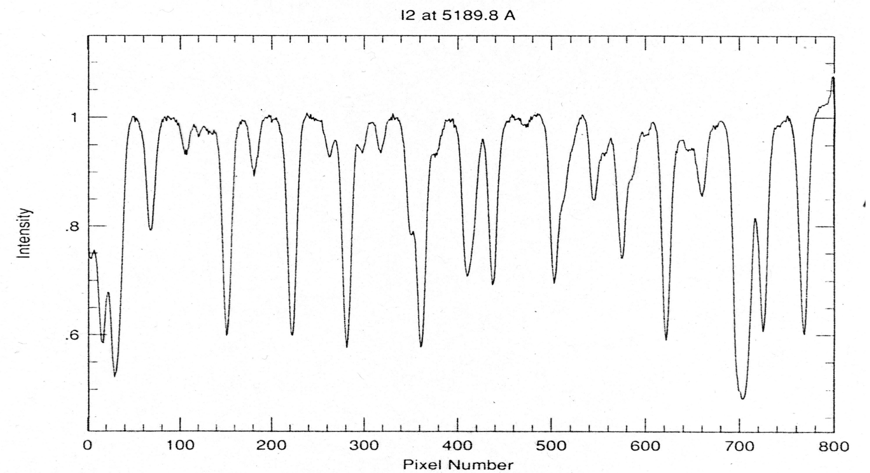
$d\lambda$ = full width of half maximum of calibration lamp emission lines



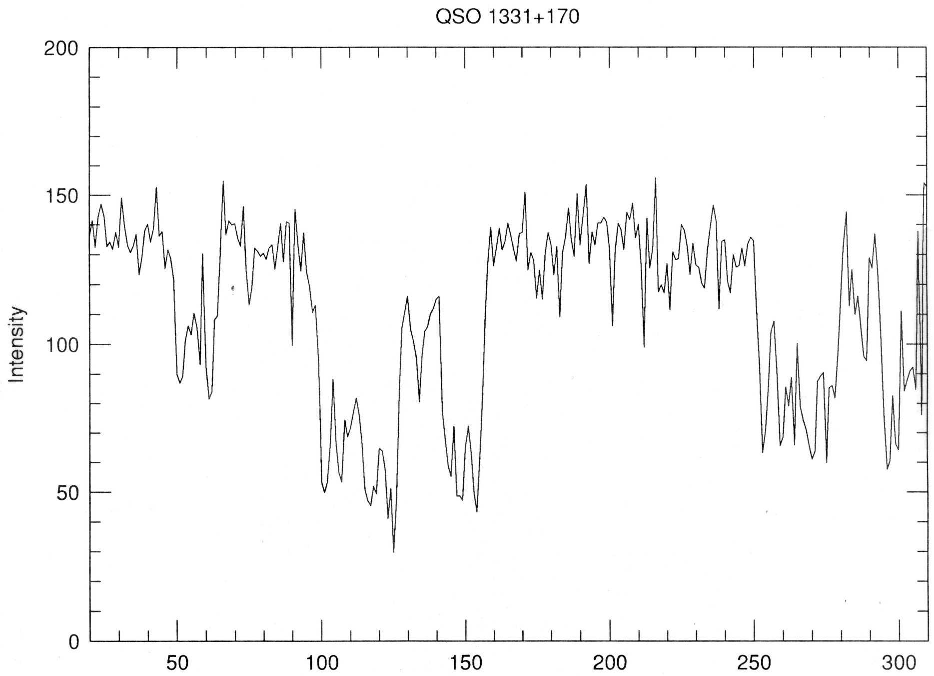
R = 15.000



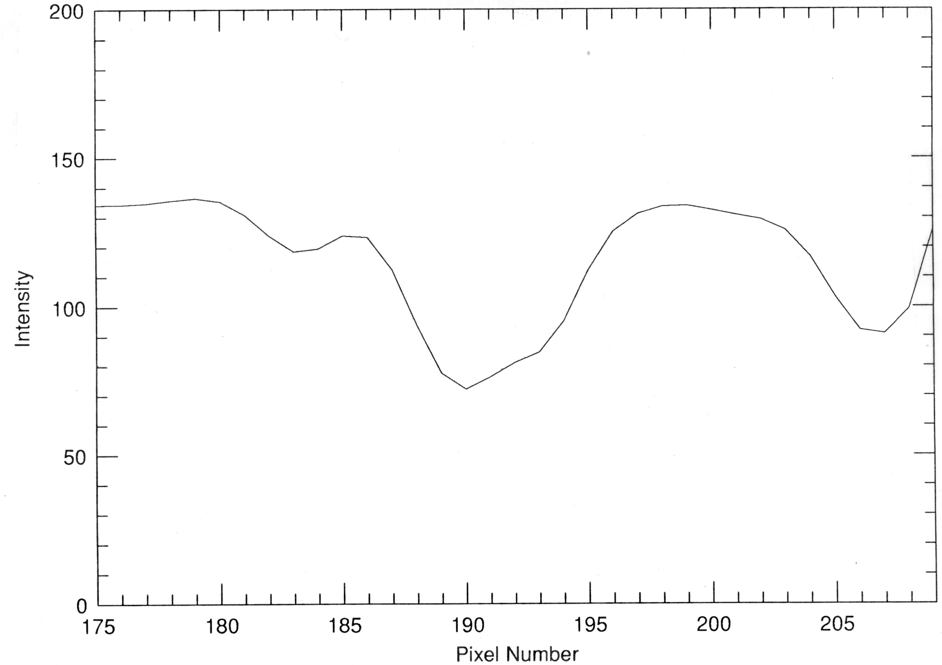
R = 100.000



R = 500.000

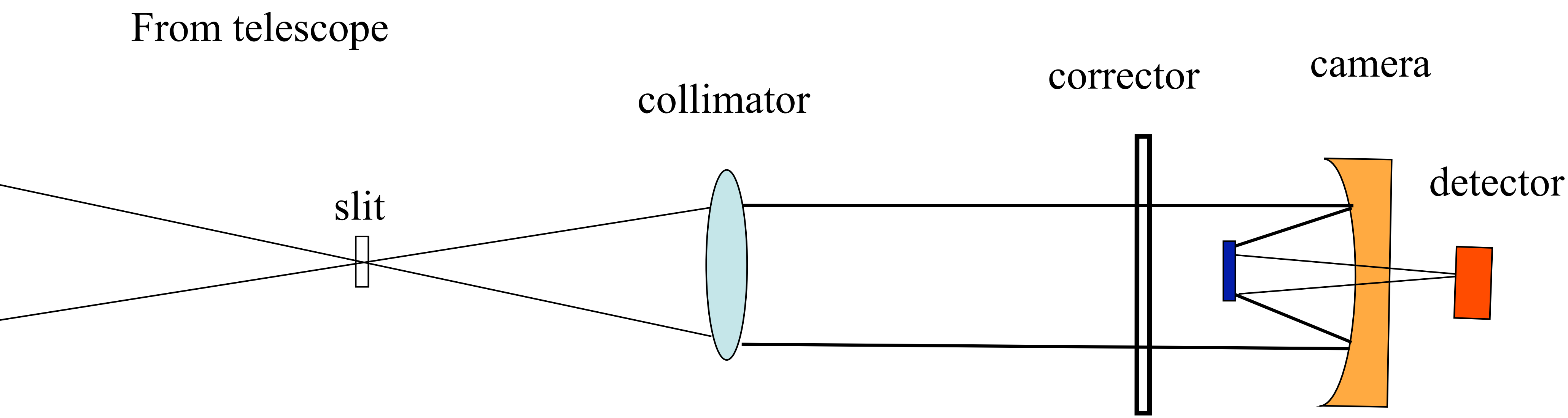


R = 30.000



R = 3.000

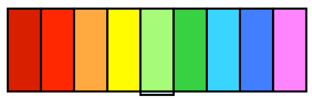
We will see that RV measurements benefit from high resolution



Without the grating a spectograph
is just an imaging camera

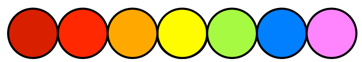
A spectrograph is just a camera which produces an image of the slit at the detector. The dispersing element produces images as a function of wavelength

slit



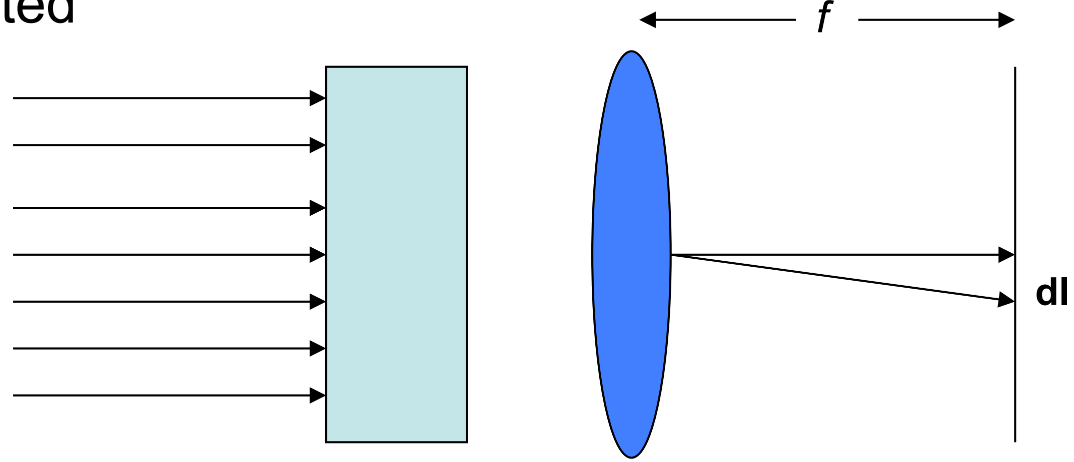
with disperser

fiber



with disperser

In collimated light



$$\frac{d\mathbf{l}}{d\lambda} = f\frac{d\beta}{d\lambda}$$

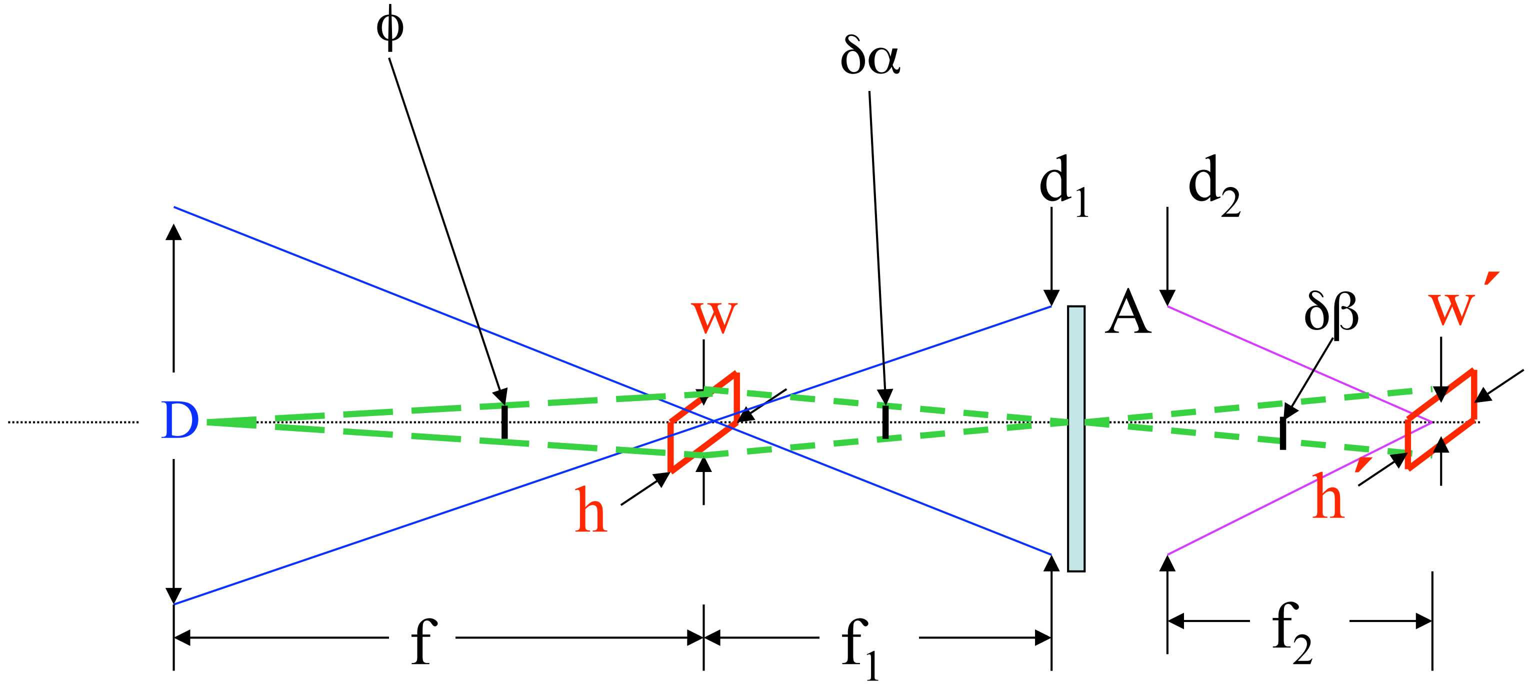
Plate Factor

$$P = (fA)^{-1} = \left(f\,\frac{d\beta}{d\lambda}\right)^{-1}$$

A is your dispersion

P is in Angstroms/mm

P x CCD pixel size = Ang/pixel



D = Diameter of telescope

d_1 = Diameter of collimator

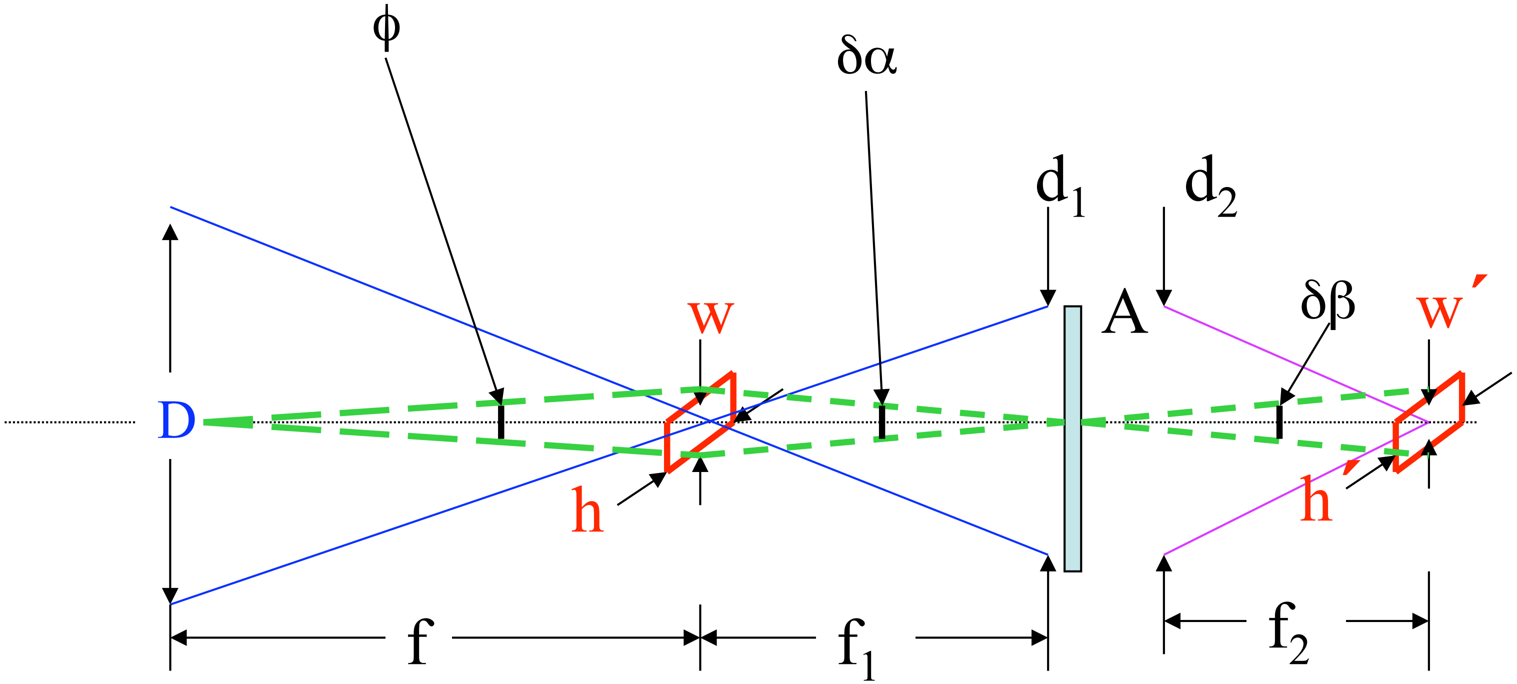
d_2 = Diameter of camera

f = Focal length of telescope

f_1 = Focal length of collimator

f_2 = Focal length of camera

A = Dispersing element



w = slit width

h = slit height

Entrance slit subtends an angle ϕ and ϕ´ on the sky:

$\phi = w/f$

$\phi' = h/f$

Entrance slit subtends an angle δα and δα´ on the collimator:

$\delta\alpha = w/f_1$

$\delta\alpha' = h/f_1$

$$\boxed{\begin{array}{l} w' = rw(f_2/f_1) = r\phi DF_2 \\ h' = h(f_2/f_1) = \phi' DF_2 \end{array}}$$

$F_2 = f_2/d_1$

r = anamorphic magnification due to dispersing element = d_1/d_2

$$\boxed{w' = rw(f_2/f_1) = r\phi DF_2}$$

This expression is important for matching slit to detector:
$2\Delta = r\phi DF_2$ for Nyquist sampling (2 pixel projection of slit).
1 CCD pixel (Δ) typically 15 – 20 μm

Example 1:

$\phi = 1$ arcsec, $D = 2$m, $\Delta = 15\mu$m $\Rightarrow rF_2 = 3.1$

Example 2:

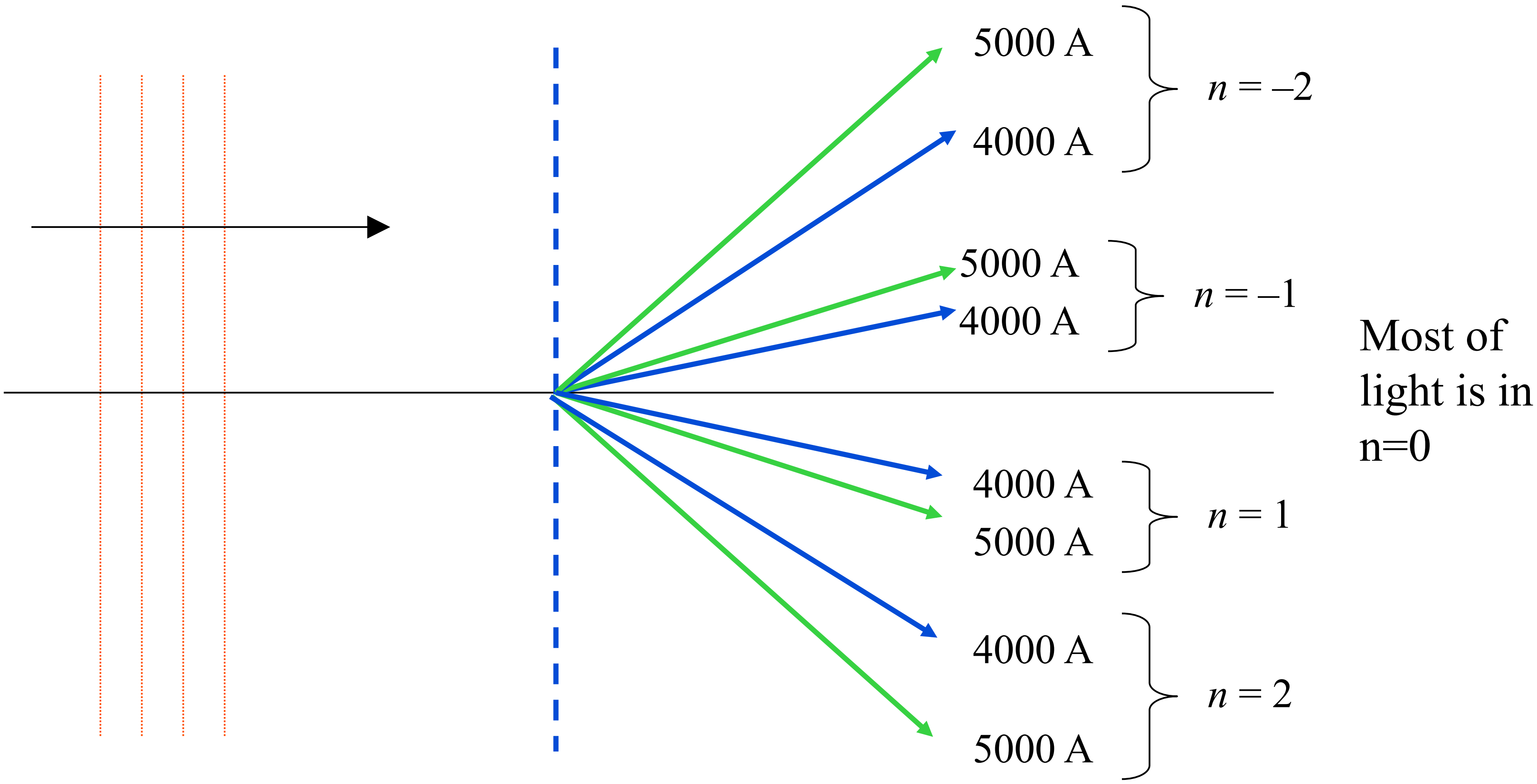
$\phi = 1$ arcsec, $D = 4$m, $\Delta = 15\mu$m $\Rightarrow rF_2 = 1.5$

Example 3:

$\phi = 0.5$ arcsec, $D = 10$m, $\Delta = 15\mu$m $\Rightarrow rF_2 = 1.2$

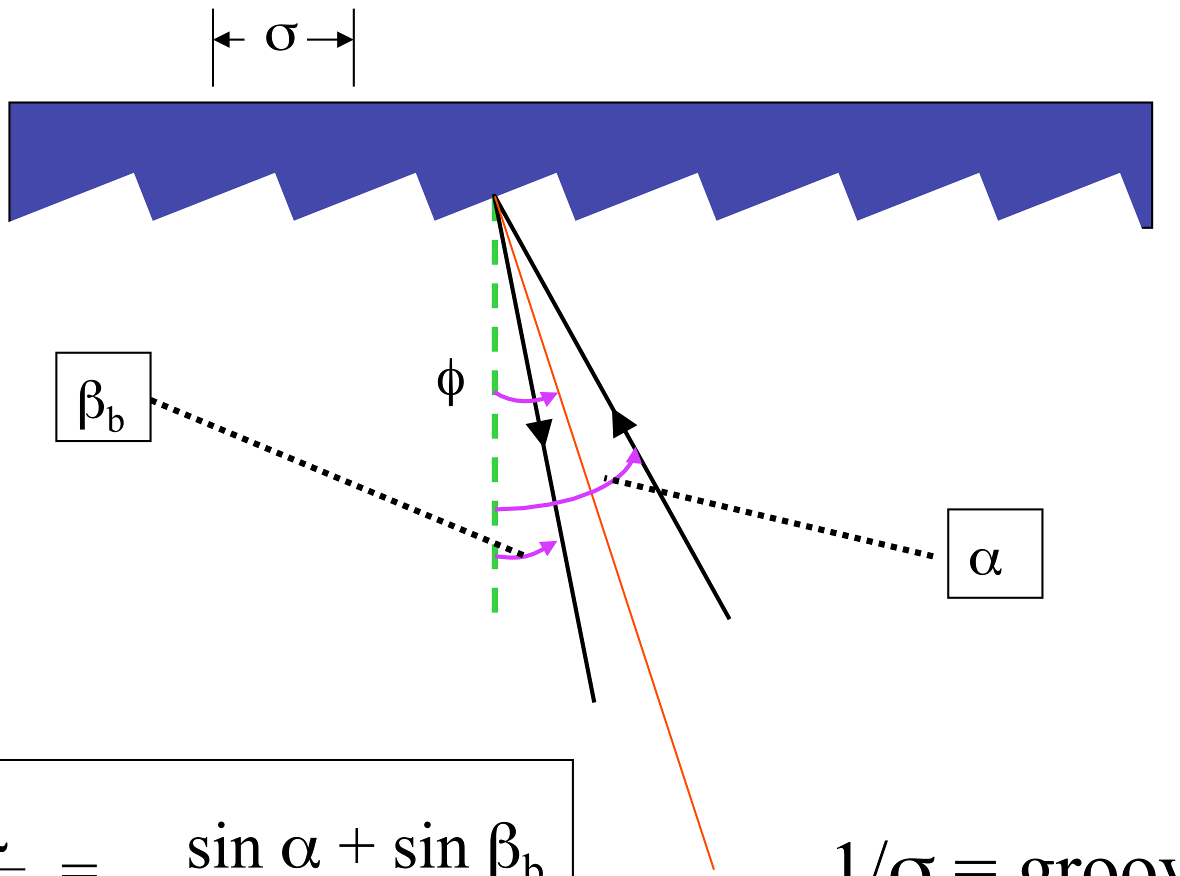
Example 4:

$\phi = 0.1$ arcsec, $D = 100$m, $\Delta = 15\mu$m $\Rightarrow rF_2 = 0.6$



5000 A
4000 A
n = –2
5000 A
4000 A
n = –1
Most of light is in n=0
4000 A
5000 A
n = 1
4000 A
5000 A
n = 2

The Grating Equation



$$\frac{m\lambda}{\sigma} = \sin\alpha + \sin\beta_b$$

$1/\sigma$ = grooves/mm

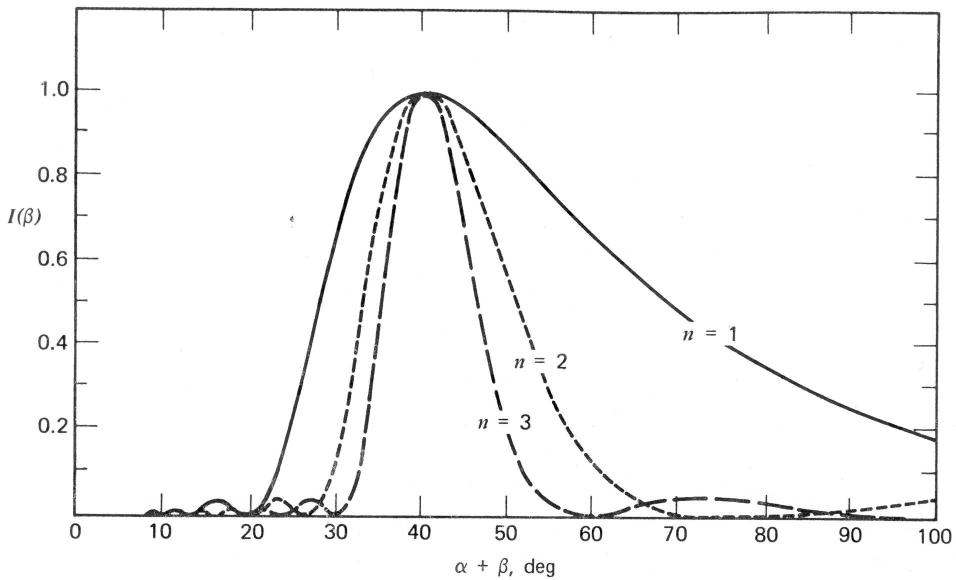


Fig. 3.9. Eq. (3.11) gives this scalar-wave blaze distribution of light. The first three orders are shown for $\phi = 20°$.

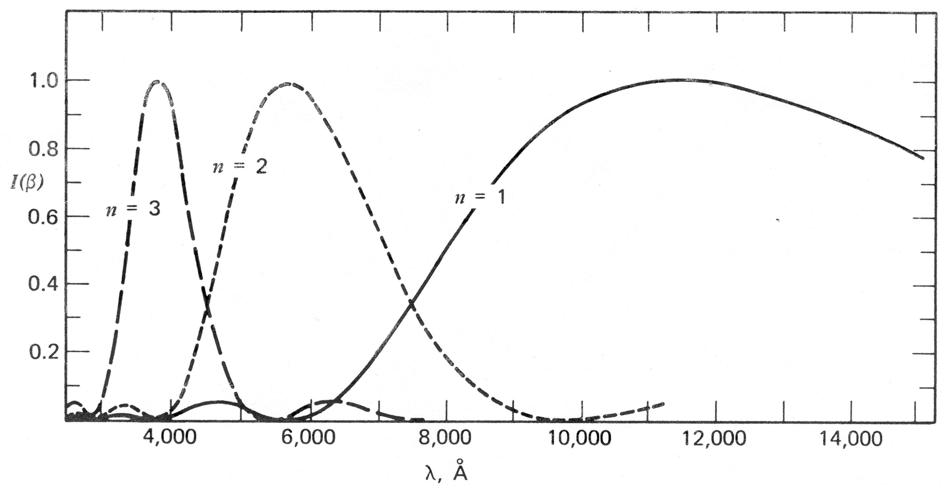


Fig. 3.10. Here the blaze distributions of Fig. 3.9 are plotted on a wavelength scale using $\alpha = \phi - 20°$.

Angular Dispersion:

$$\frac{d\beta}{d\lambda} = \frac{m}{\sigma \cos \beta} = \frac{\sin \alpha + \sin \beta}{\lambda \cos \beta}$$

Linear Dispersion:

$$dx = f_{cam}\, d\beta$$

$$\frac{d\lambda}{dx} = \frac{d\lambda}{d\beta}\,\frac{d\beta}{dx} = \frac{1}{f_{cam}}\,\frac{1}{d\beta/d\lambda} \quad \text{Angstroms/mm}$$

Resolving Power

$$dx = f_2 \frac{d\beta}{d\lambda} \Delta\lambda$$

$$w' = rw(f_2/f_1) = r\phi D F_2$$

$$f_2 \frac{d\beta}{d\lambda} \Delta\lambda = r\phi D F_2$$

Recall: $F_2 = f_2/d_1$

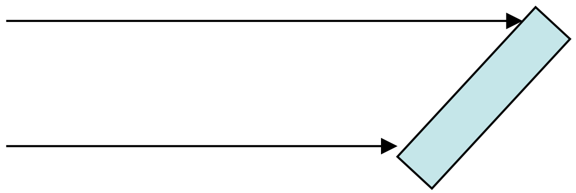
$$\delta\lambda = \frac{r\phi}{A} \frac{D}{d_1}$$

For a given telescope R depends only on collimator diameter

$$R = \lambda/d\lambda = \frac{\lambda A}{r} \frac{1}{\phi} \frac{d_1}{D}$$

$R = 100.000$ $A = 1.7 \times 10^{-3}$

D(m)	ϕ (arcsec)	d_1 (cm)
2	1	10
4	1	20
10	1	52
10	0.5	26
30	0.5	77
30	0.25	38



What if adaptive optics can get us to the diffraction limit?

Slit width is set by the diffraction limit:

$$\phi = \frac{\lambda}{D}$$

$$R = \frac{\cancel{\lambda}}{r} A \frac{\cancel{D}}{\cancel{\lambda}} \frac{d_1}{\cancel{D}} = \frac{A}{r} d_1$$

R	d_1
100000	0.6 cm
1000000	5.8 cm

Normal gratings:

- ruling 600-1200 grooves/mm
- Used at low blaze angle (~10-20 degrees)
- orders m=1-3

Echelle gratings:

- ruling 32-80 grooves/mm
- Used at high blaze angle (~65 degrees)
- orders m=50-120

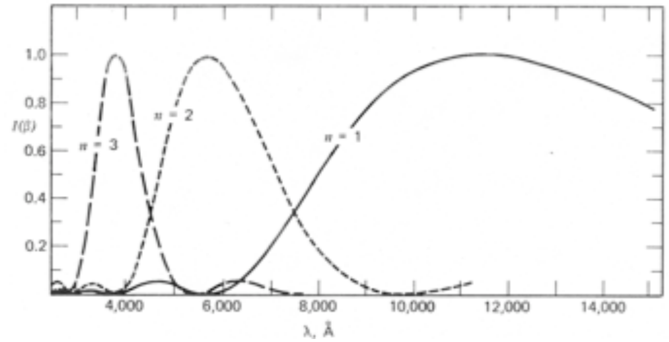
242 **13. Dispersing Elements and Systems**

	m	$1/\sigma$ (mm)	δ (°)
First-order grating	1	1200	17.5
Echelle	45	79	63.5

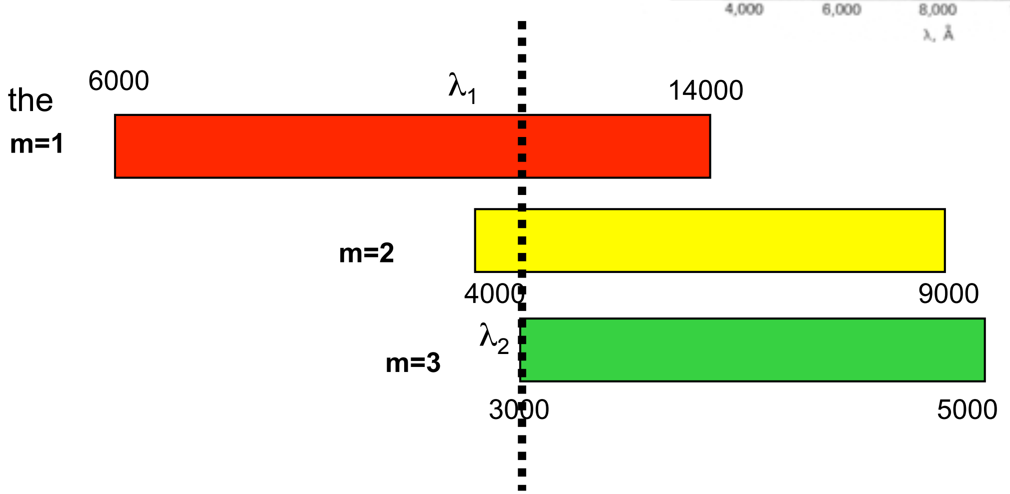
$\alpha = \delta + \theta, \quad \beta = \delta - \theta, \quad \theta > 0 \quad \text{if} \quad \alpha > \beta$

Both satisfy grating equation for λ = 5000 A

1200 gr/mm grating



Schematic: orders separated in the vertical direction for clarity

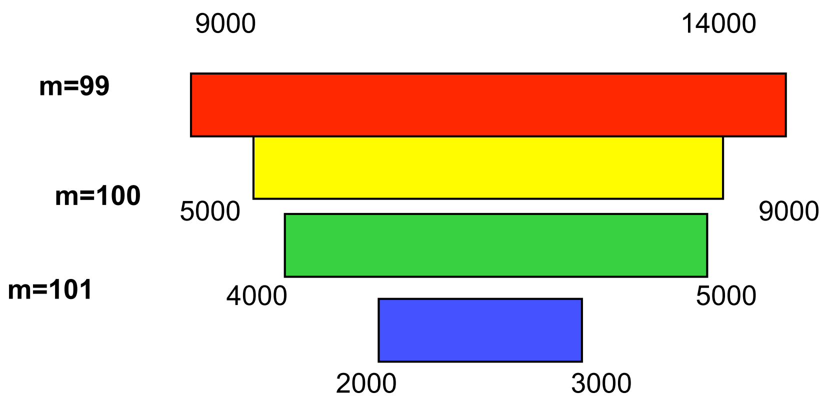


You want to observe λ_1 in order m=1, but light λ_2 at order m=2, where $\lambda_1 \neq \lambda_2$ contaminates your spectra

Order blocking filters must be used

79 gr/mm grating

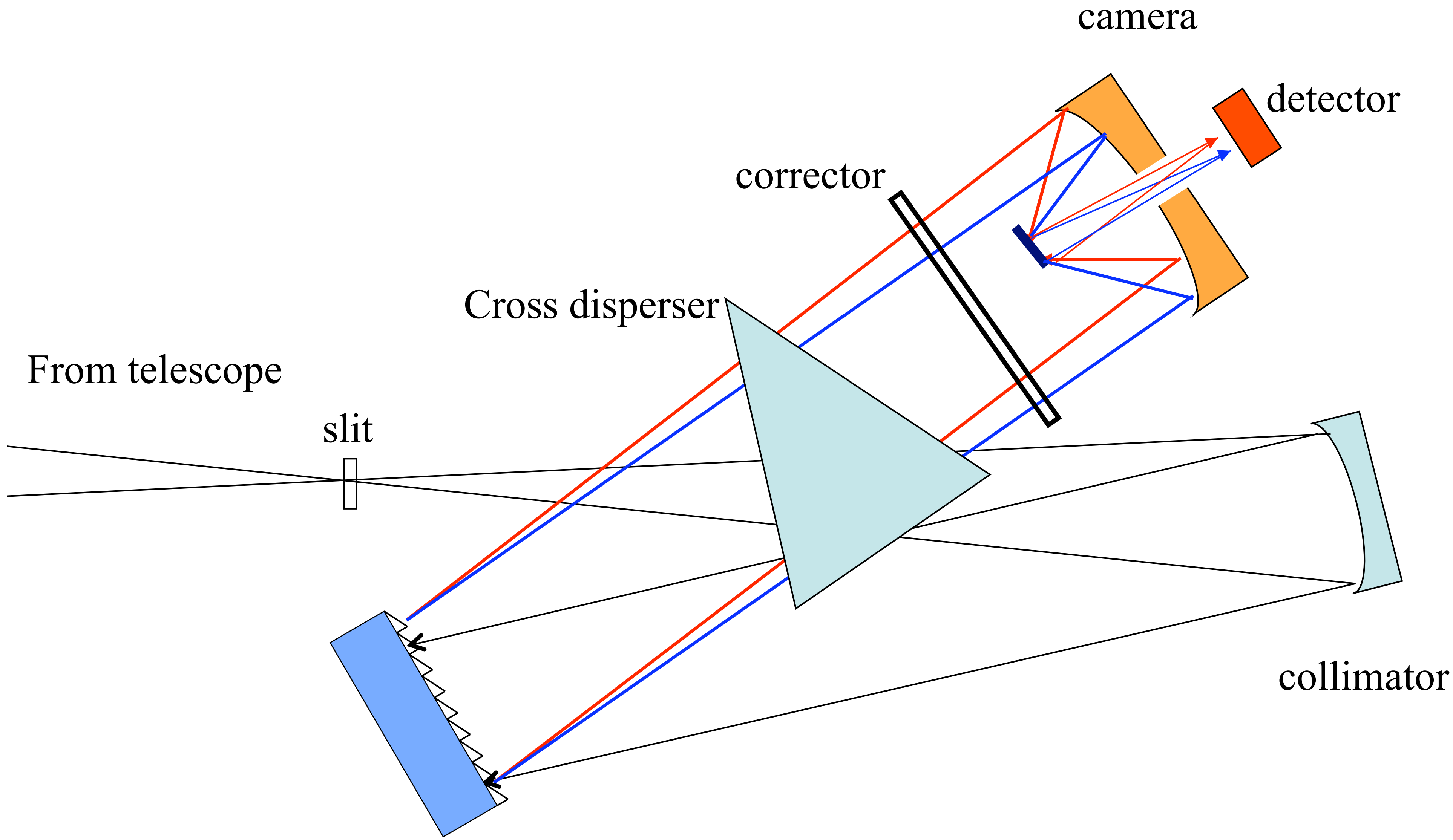
Schematic: orders separated in the vertical direction for clarity

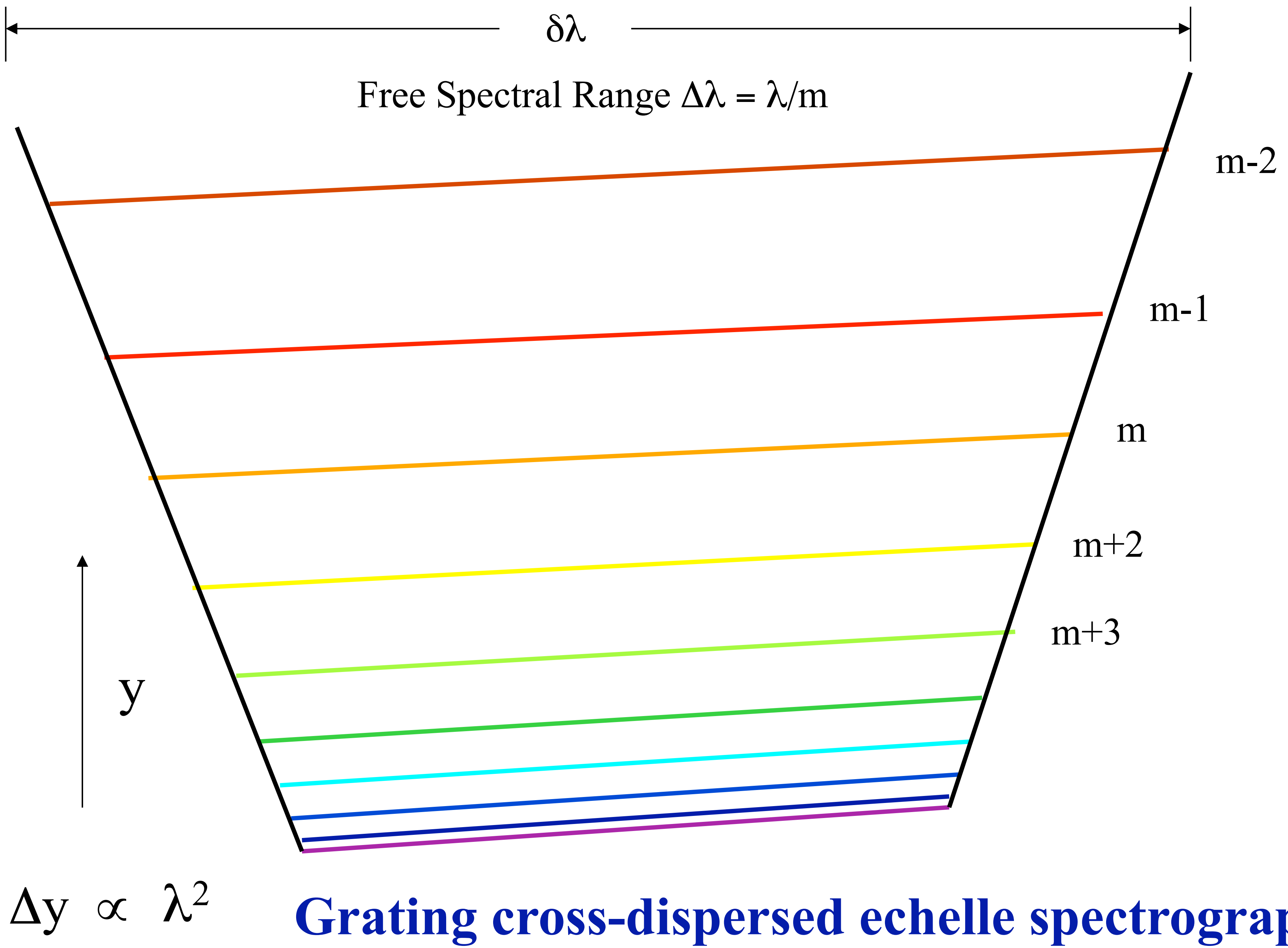


In reality:

Need interference filters but why throw away light?

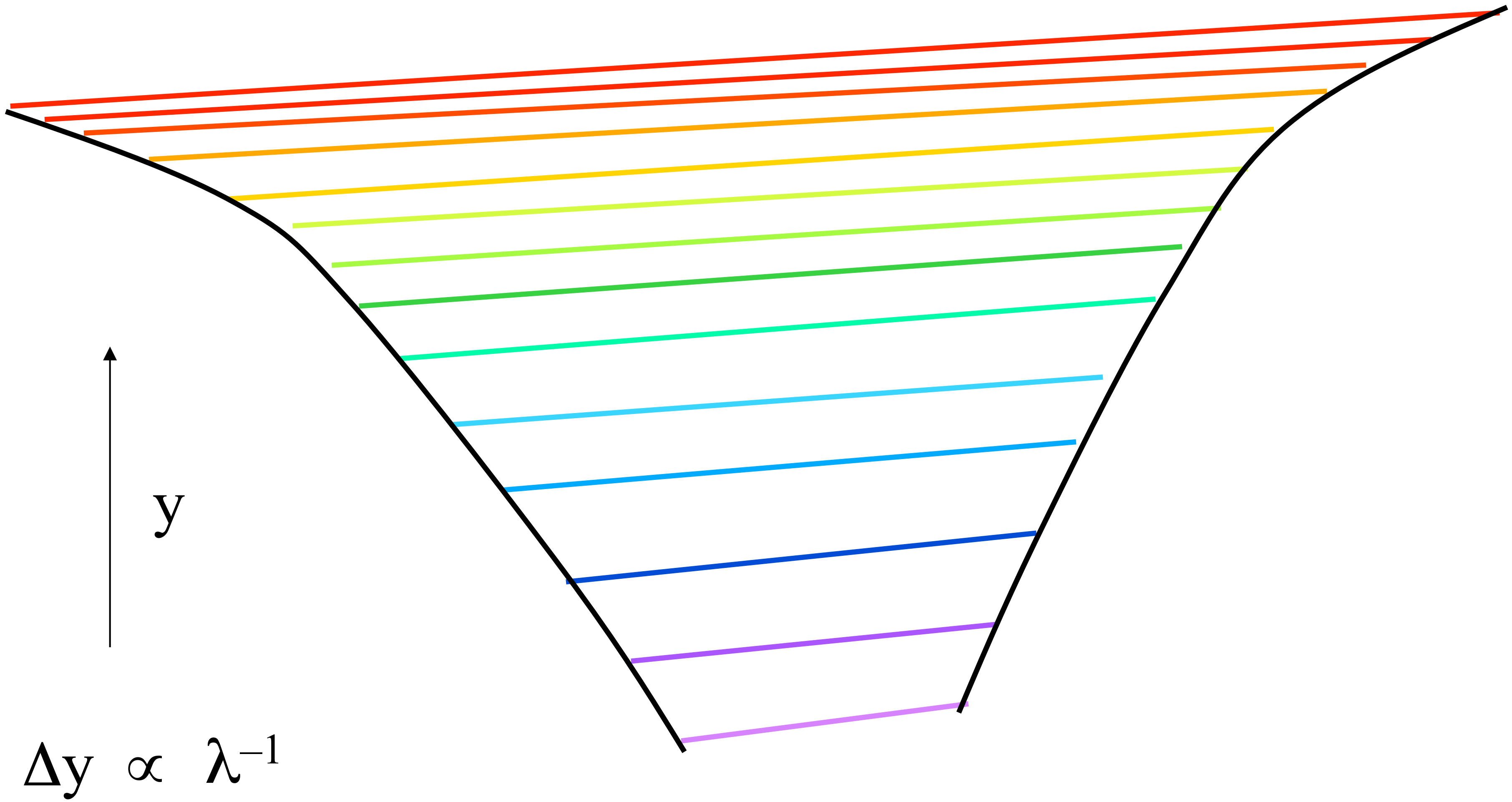
Spectrographs



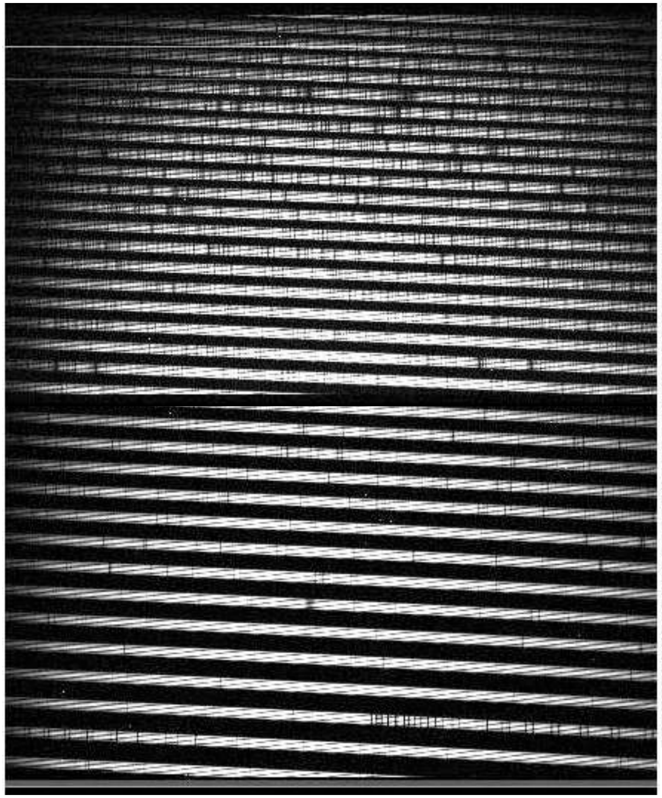


Grating cross-dispersed echelle spectrographs

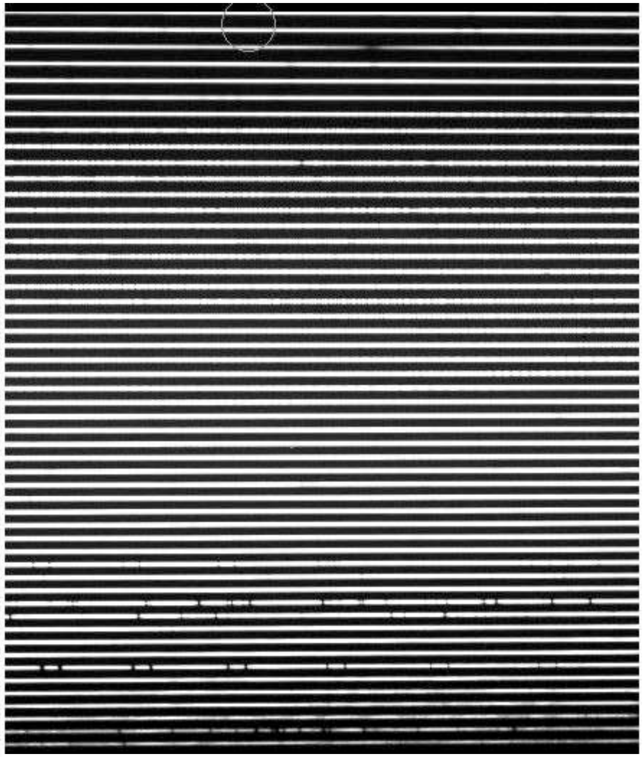
Prism cross-dispersed echelle spectrographs



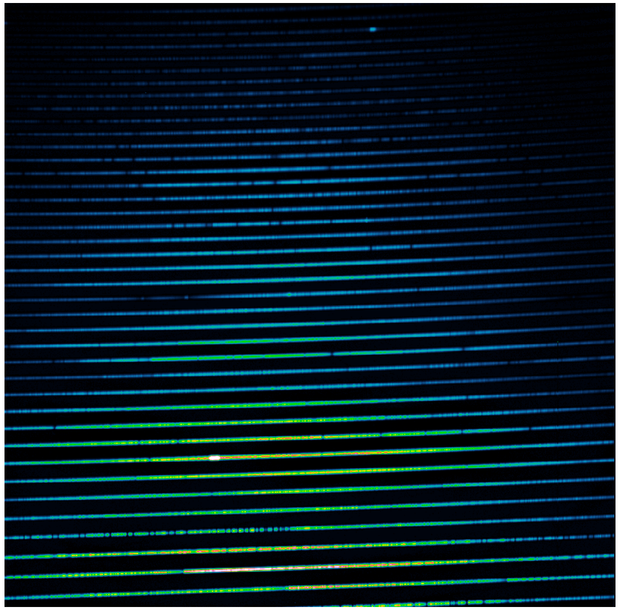
Cross dispersion



grating



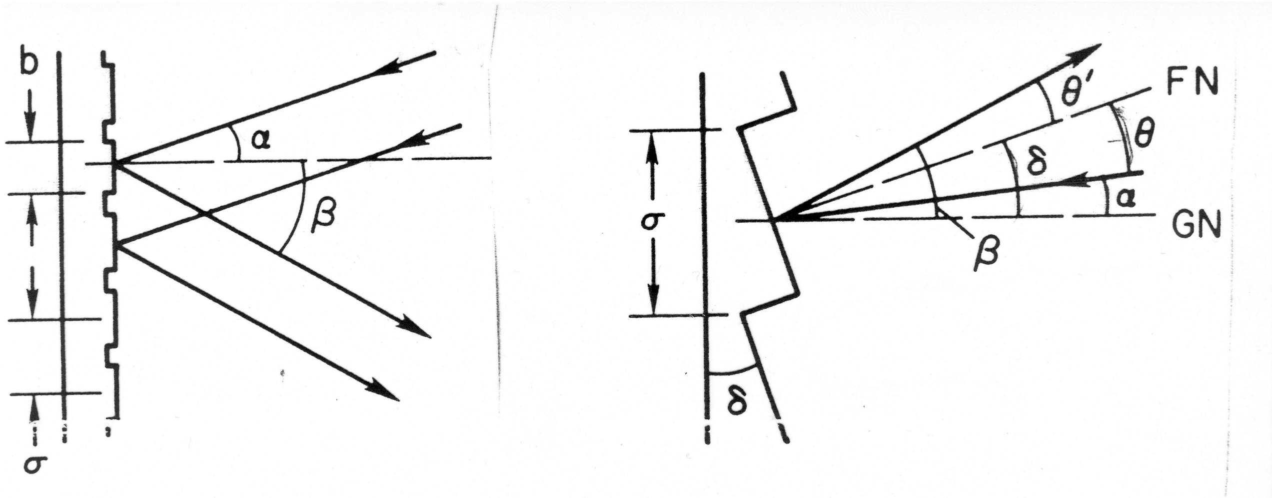
prism



grism

$$\Delta y \propto \lambda^2 \cdot \lambda^{-1} = \lambda$$

Increasing wavelength



<u>Blaze Function:</u>

$$I = IF \times BF = \frac{sin^2 N\gamma'}{sin^2\gamma'}\frac{sin^2\gamma}{\gamma^2}$$

$2\gamma'$ = phase difference between center of adjacent grooves

γ = phase diference between center and edge of one groove.

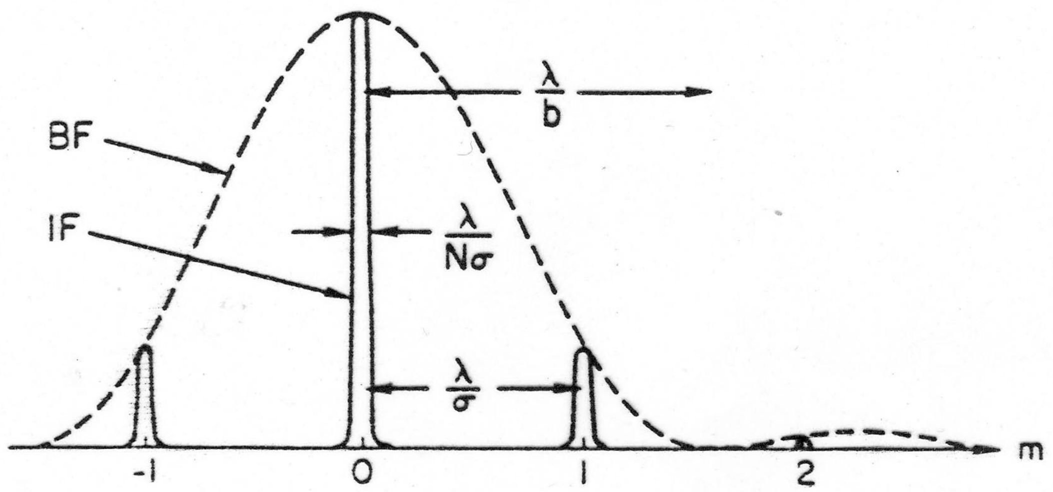
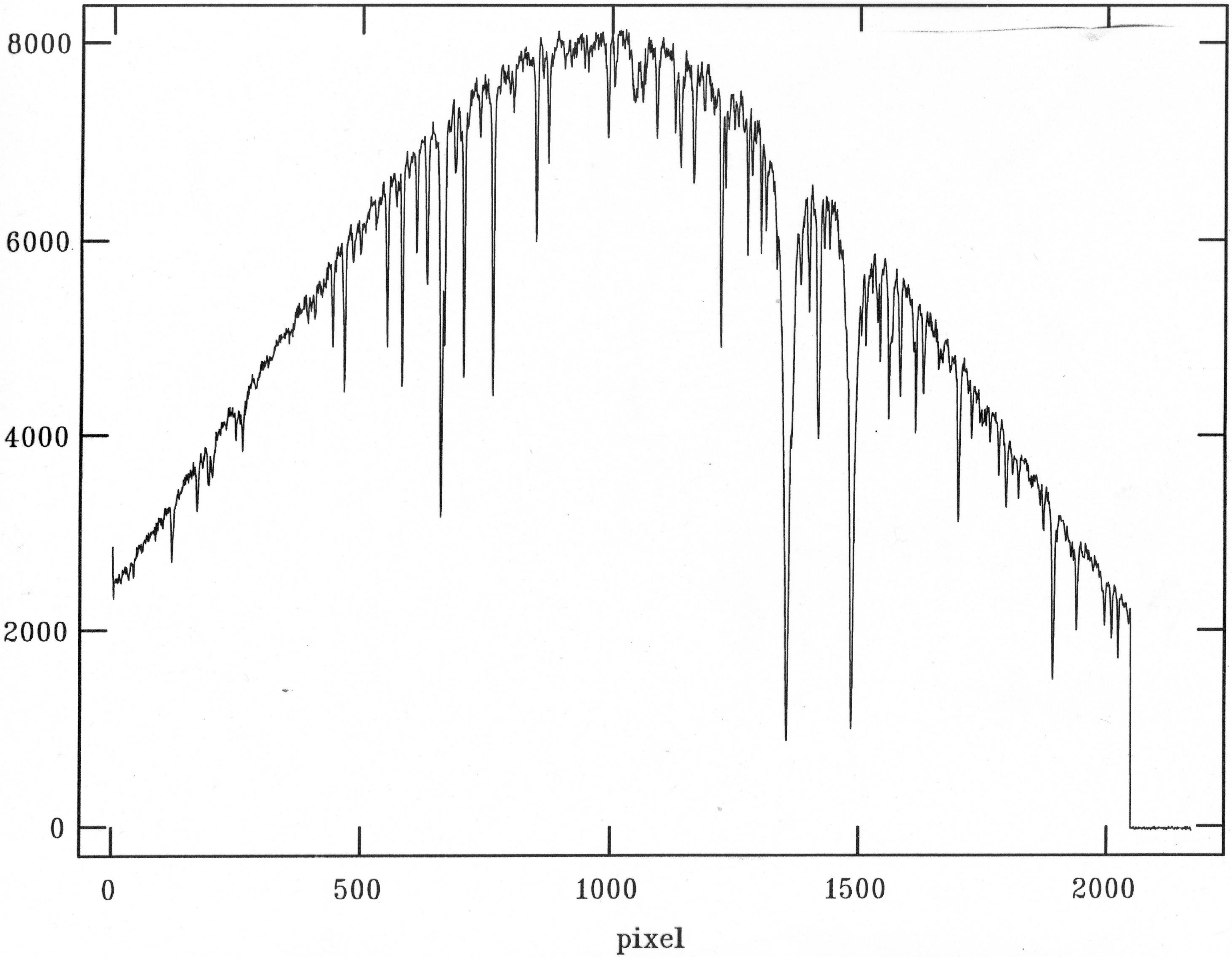
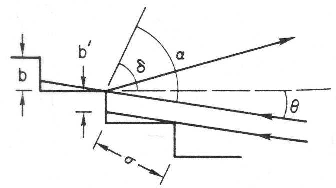


Fig. 13.6. Intensity pattern of single diffracted wavelength for grating in Fig. 13.5. BF, Blaze function; IF, interference factor.


8000
6000
4000
2000
0
0
500
1000
1500
2000
pixel

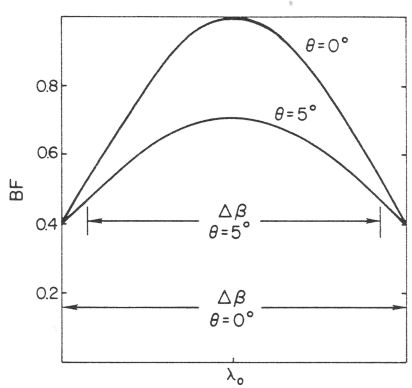
$\theta \neq 0$:

Blaze function is determined by the effective width of a single groove $\Rightarrow$ depends on θ.



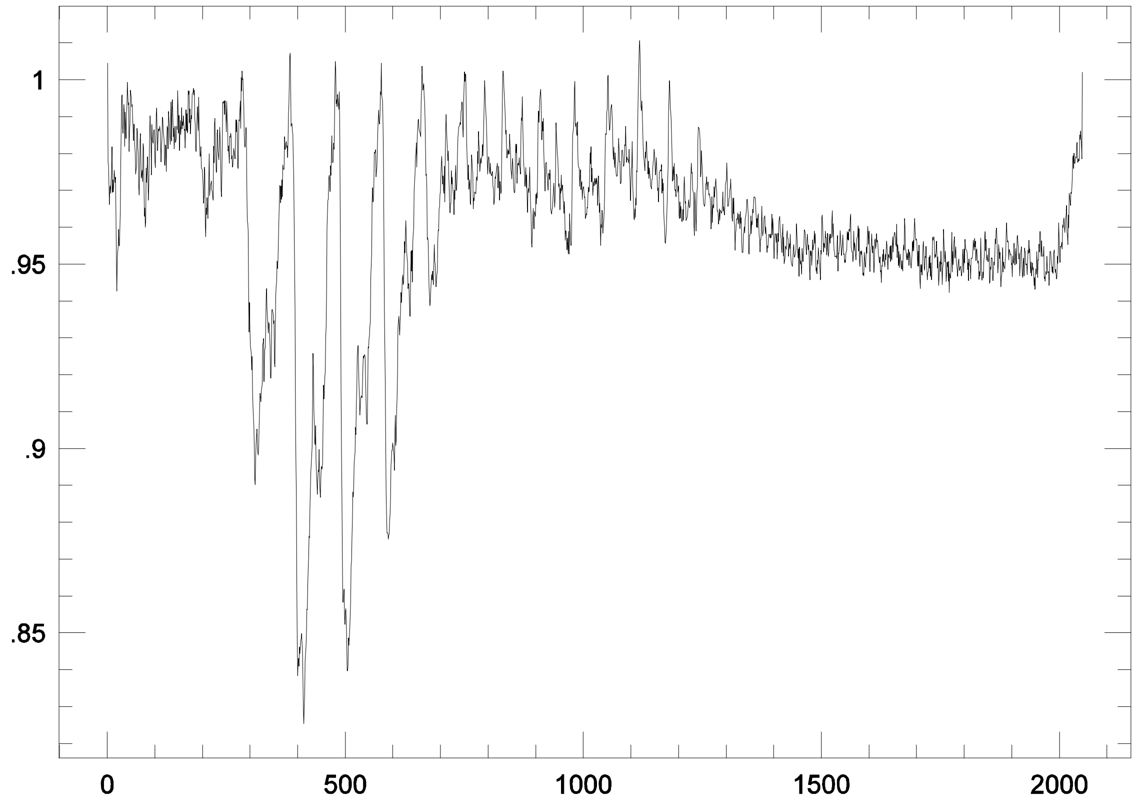
$b' = \sigma cos\alpha / cos\beta$

III. Grating Efficiency

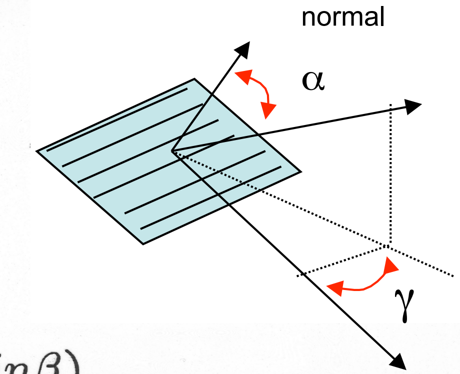


Choice of θ depends on mechanical constraints, desired efficiency, shape of blaze function, and.....the "picket fence".

- Chose θ and γ, choice depends on
  - Efficiency
  - Space constraints
  - „Picket Fence“ for Littrow configuration



Line tilt:



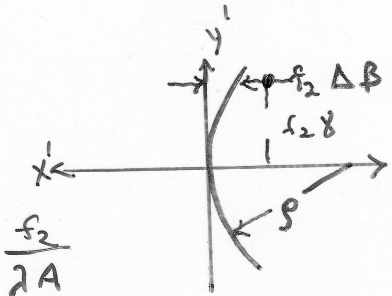
General Grating Equation:

$$\frac{m\lambda}{\sigma} = cos\gamma(sin\alpha + sin\beta)$$

$$\frac{d\beta}{d\gamma} = tan\gamma\frac{m\lambda}{\sigma cos\beta cos\gamma}$$

but: $\frac{d\beta}{d\lambda} = \frac{m}{\sigma cos\beta cos\gamma}$

$$\frac{d\beta}{d\gamma} = \left(\lambda\frac{d\beta}{d\lambda}\right)\gamma$$



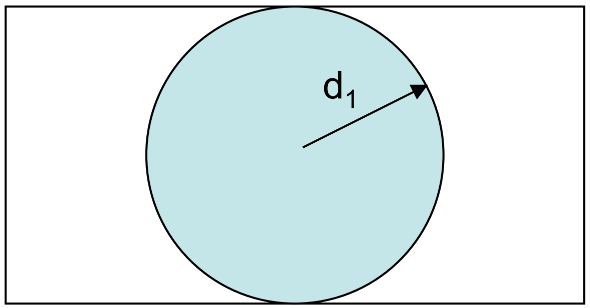
$$\rho = \frac{f_2}{\lambda A}$$

$$\Delta\beta = \lambda d\beta/d\lambda \int_0^\gamma \gamma\, d\gamma = \lambda\frac{d\beta}{d\lambda}\gamma^2/2$$

Next Lecture:

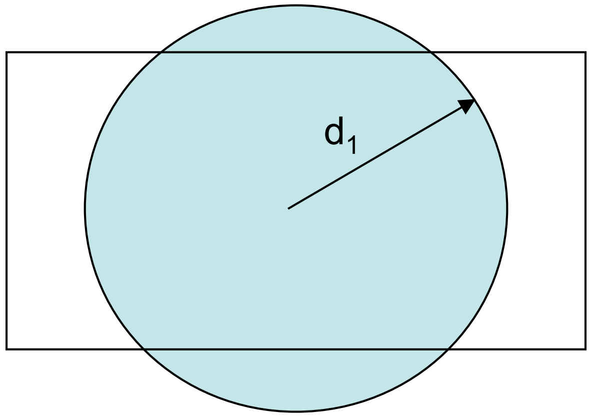
How to achieve precice radial velocity measurements (PRV)

Tricks to improve efficiency: Overfill the Echelle

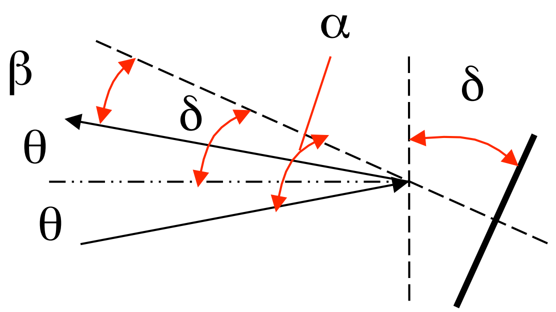


$R \sim d_1/\phi$

$w' \sim \phi/d_1$



For the same resolution you can increase the slit width and increase efficiency by 10-20%



Relation between blaze angle δ, grating normal, and angles of incidence and diffraction

Littrow configuration:

$\theta = 0,\ \alpha = \beta = \delta$

$m\,\lambda = 2\,\sigma \sin\delta$

$A = 2 \sin \delta/\lambda$

$R = 2d_1 \tan \delta/\phi\, D$

A increases for increasing blaze angle

R2 echelle, $\tan\delta = 2$, $\delta = 63.4^\circ$

R4 echelle $\tan\delta = 4$, $\delta = 76^\circ$

At blaze peak $\alpha + \beta = 2\delta$

$m\lambda_b = 2\,\sigma \sin\delta \cos\theta$

λ_b = blaze wavelength