Center for
Electronic Correlations and Magnetism
University of Augsburg

Theory of correlated fermionic condensed matter

5. Common concepts in correlated Fermi systems

XIV. Training Course in the Physics of Strongly Correlated Systems
Salerno, October 9, 2009

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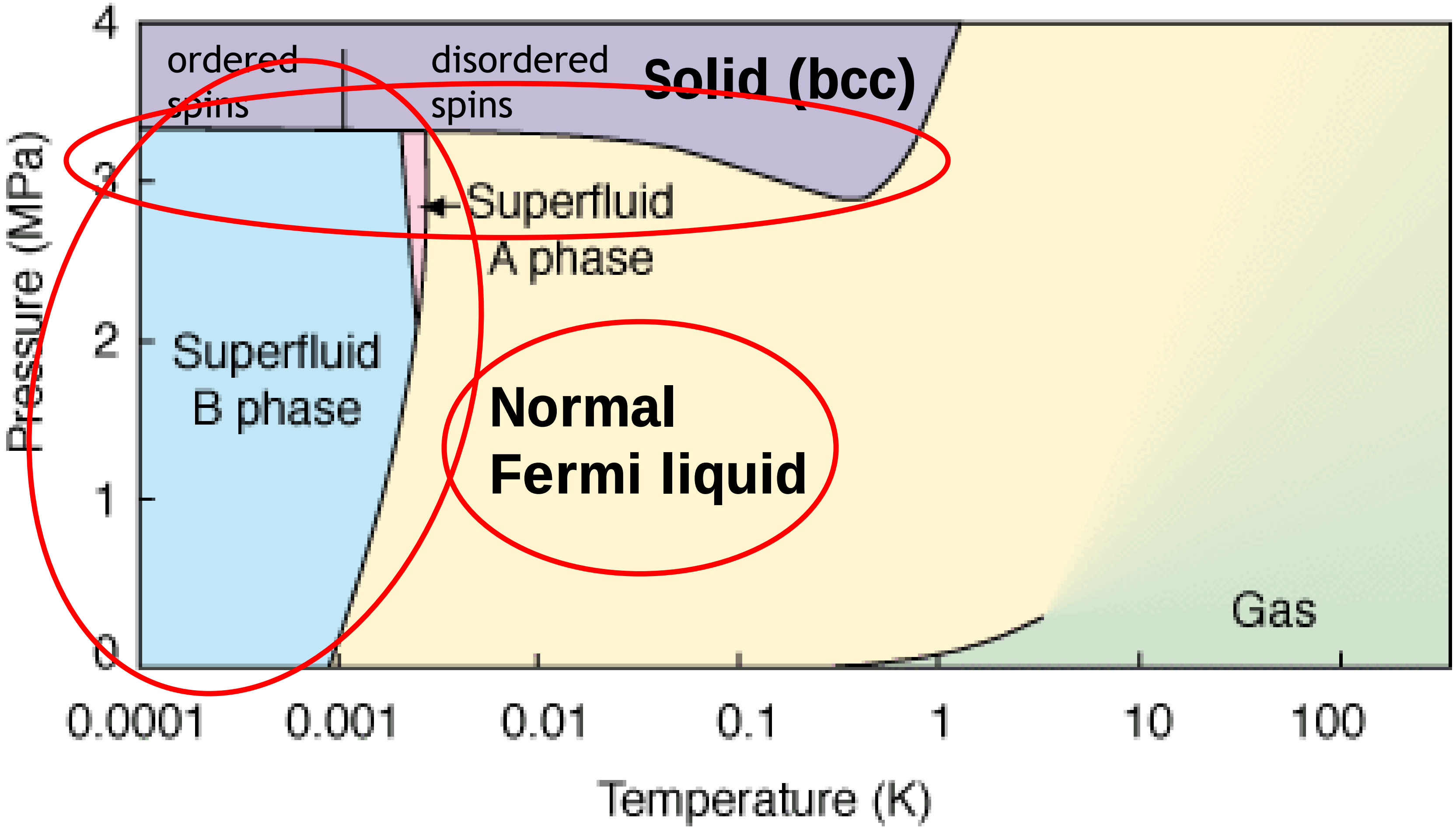
Outline:

- Common properties of correlated electrons and ^3He

- Emergence in many-particle systems

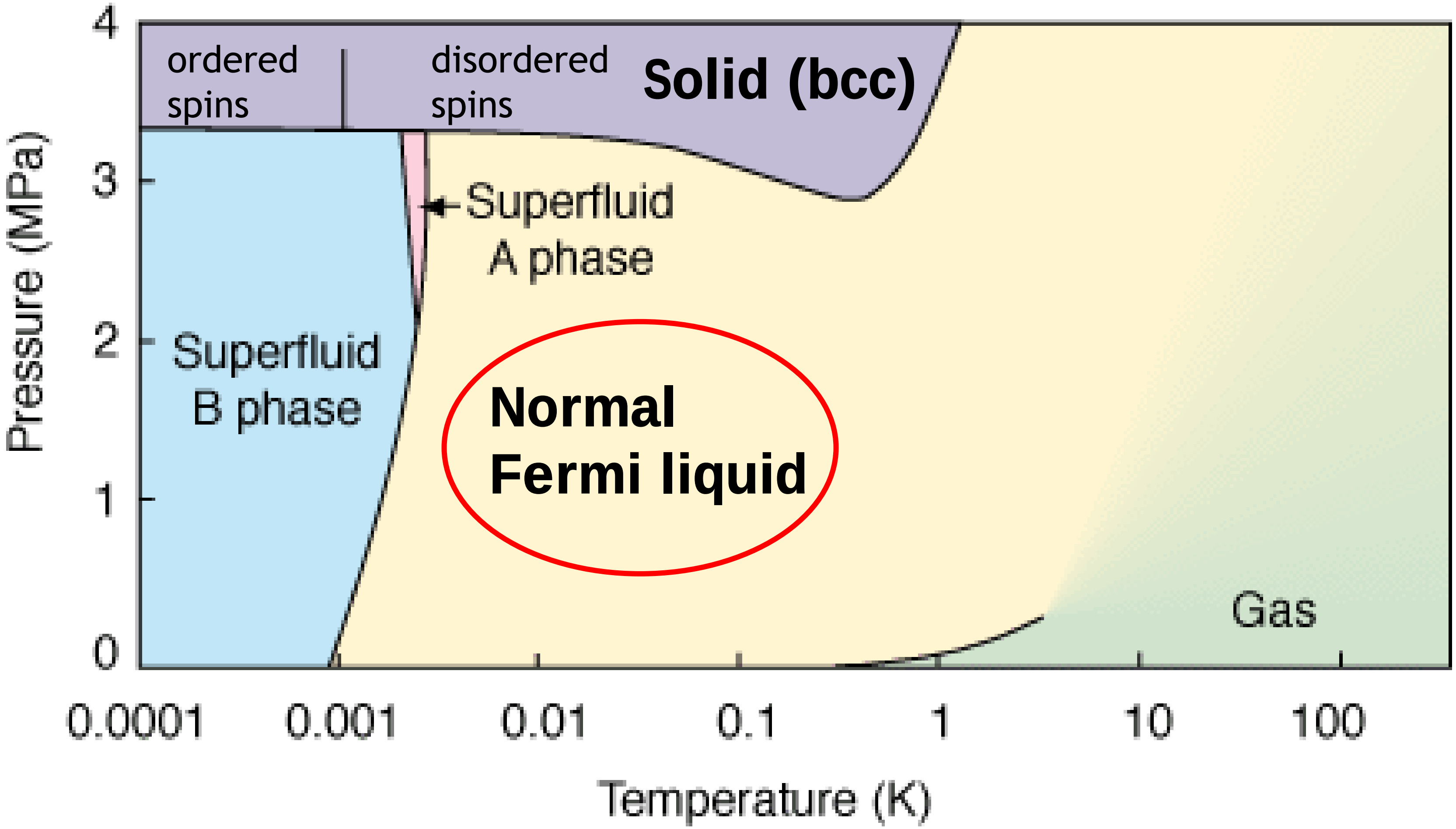
Paradigm: Helium-3

1. Normal state
2. Pair-correlated state
3. localization-delocalization transition

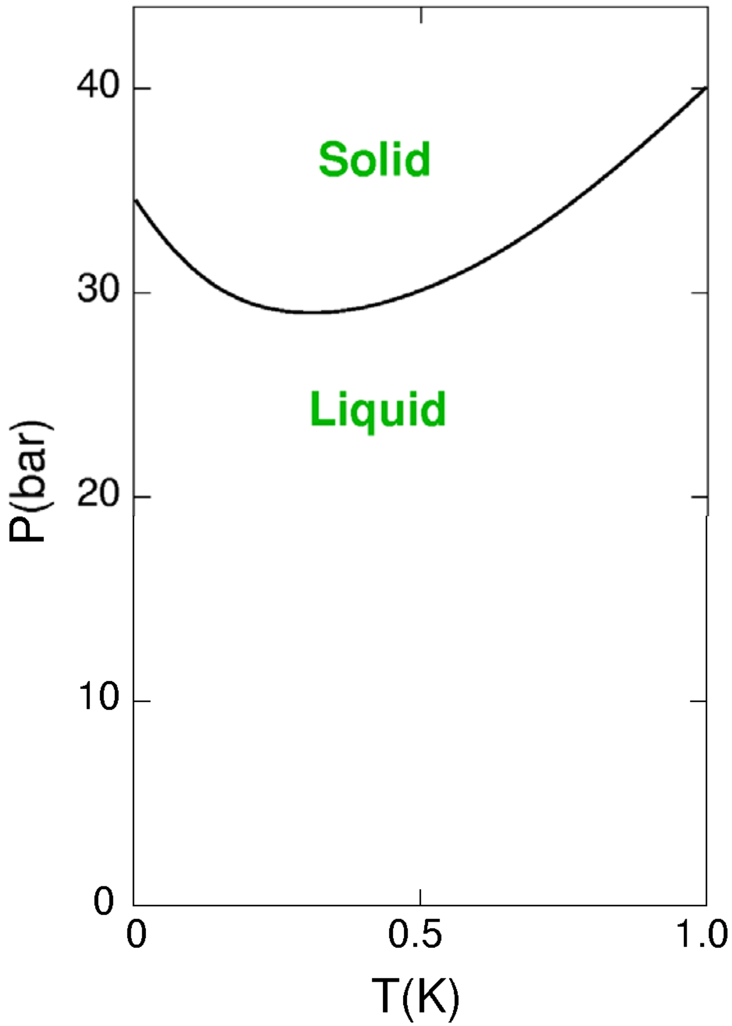


1. Normal state properties

^3He



^{3}He



Nuclear spin $I = \frac{1}{2}\hbar$

⇓

Fermion

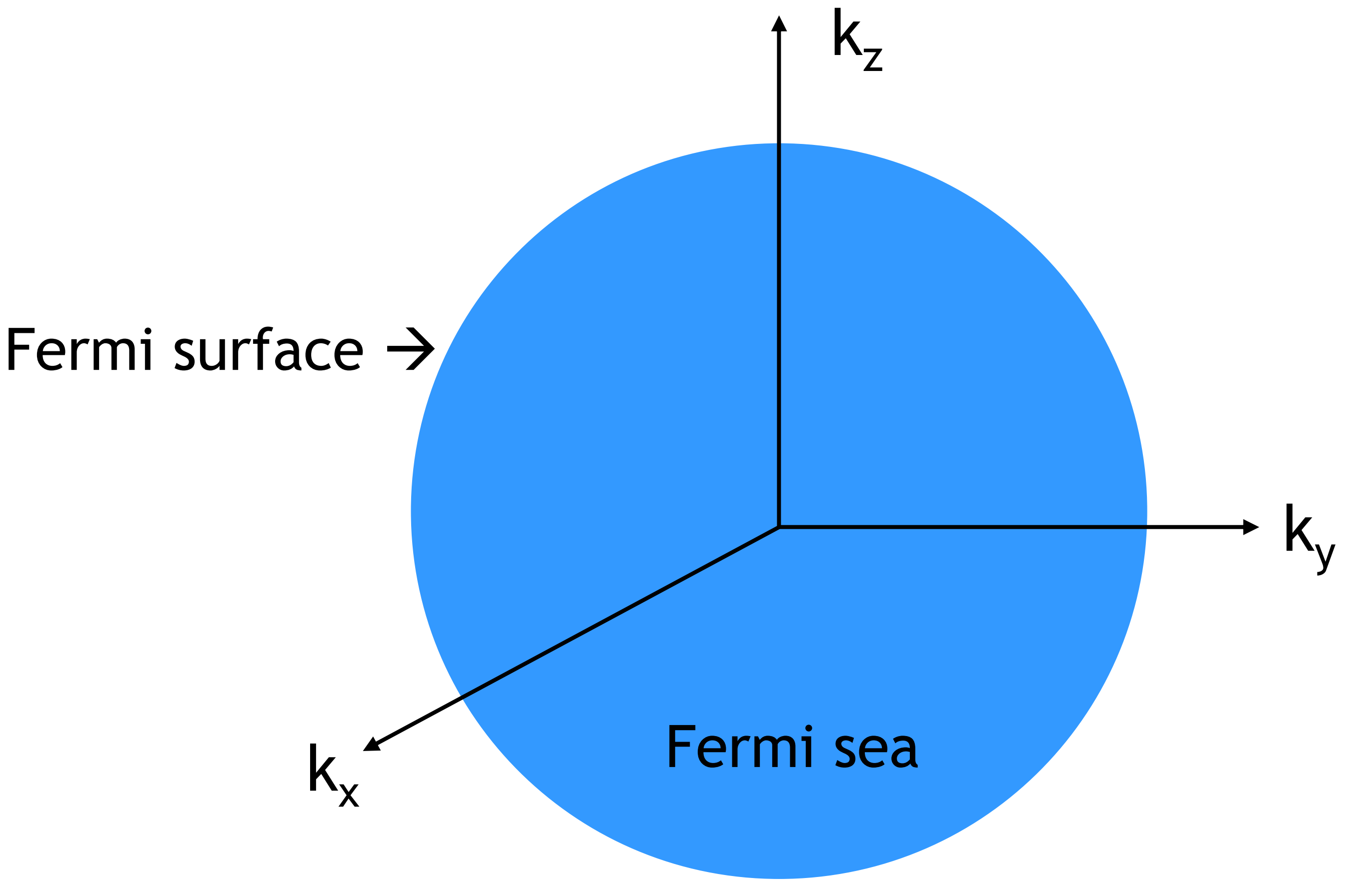
⇓

Fermi-Dirac statistics

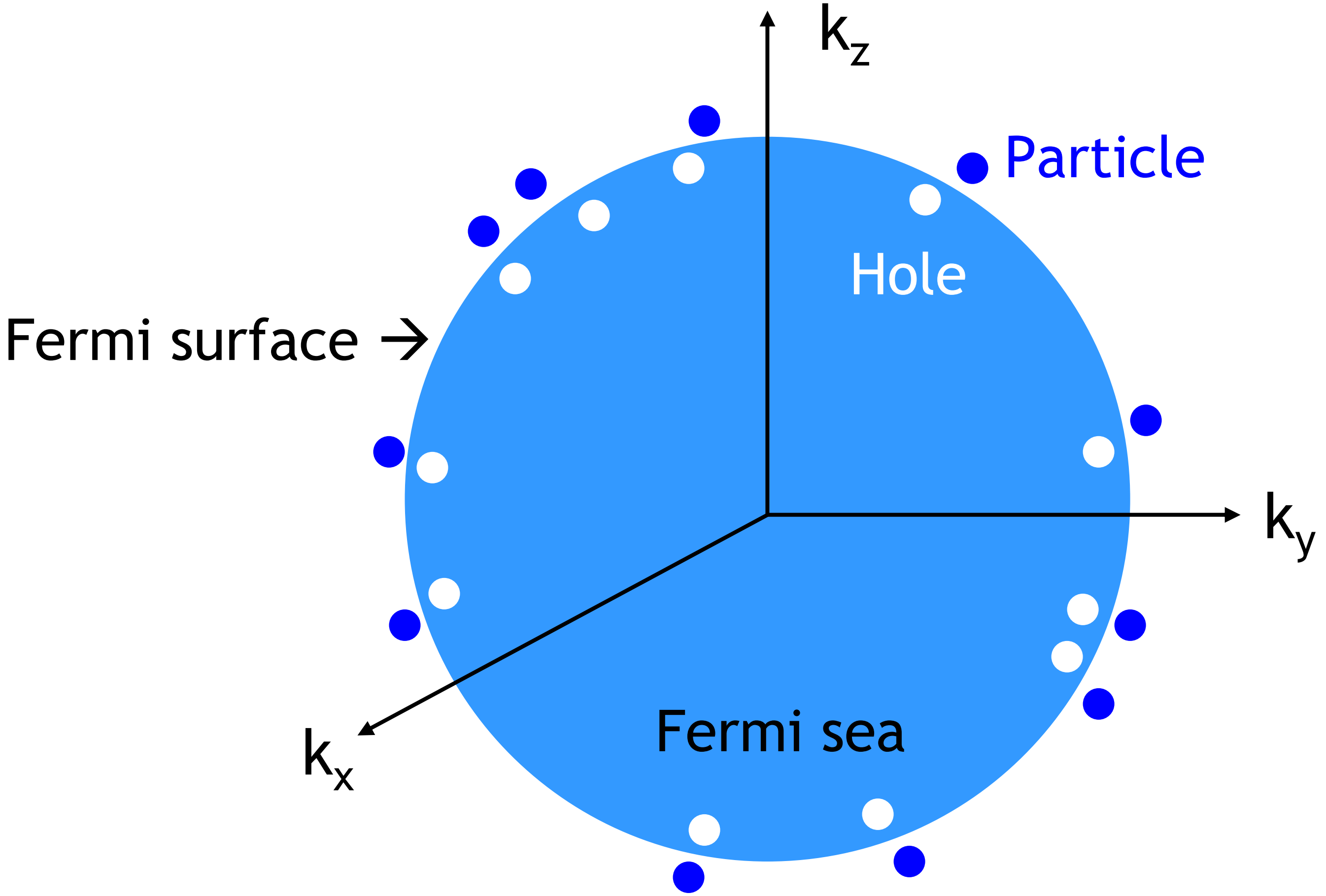
⇓

Fermi body/surface

Fermi gas: **Ground state**



Fermi gas: **Excited states (T>0)**



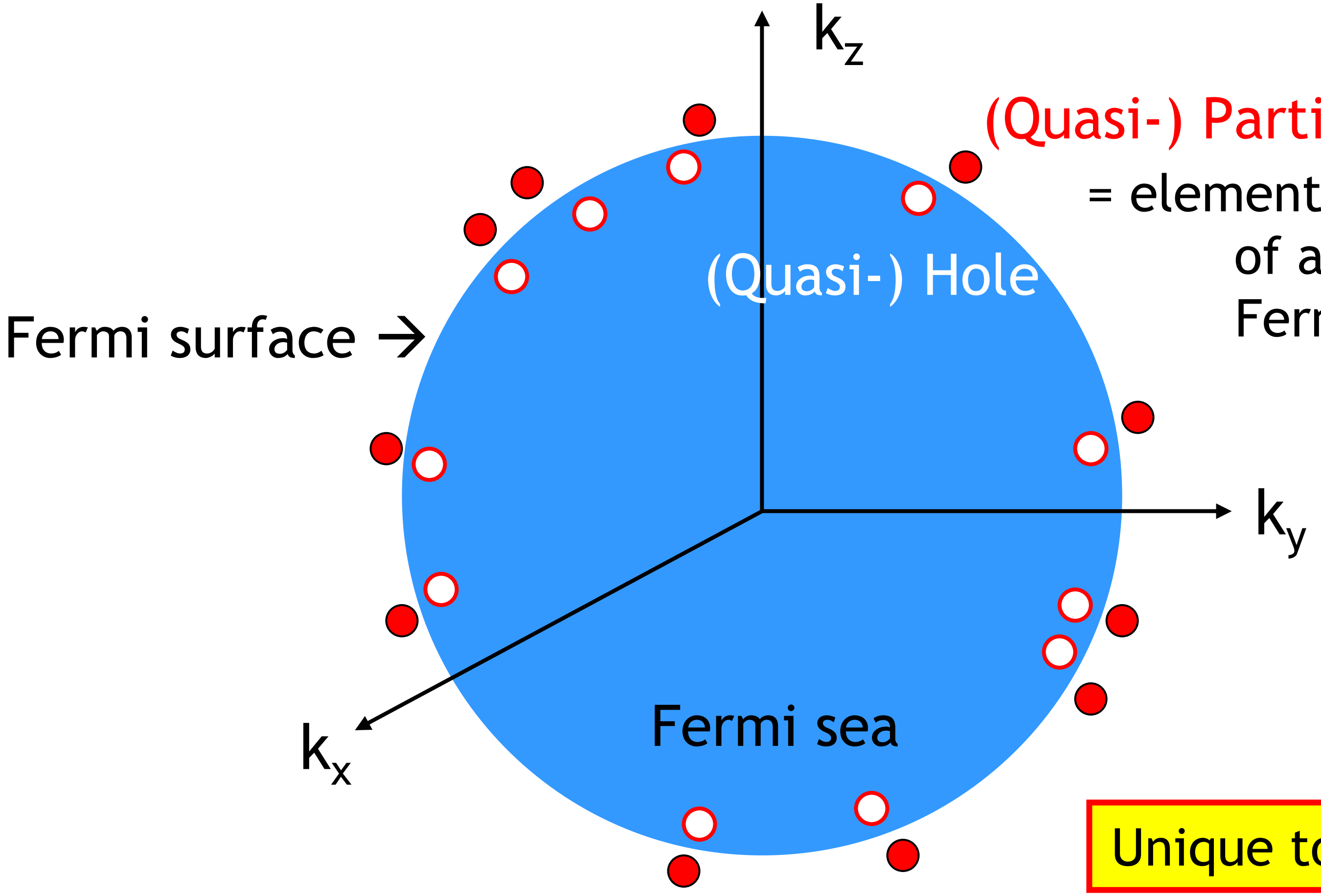
Exact **k**-states ("particles"): infinite life time

Switch on interaction adiabatically (d=3)

Landau Fermi liquid

Landau (1956/58)

1-1 correspondence between **k**-states



= elementary excitation of an interacting Fermi system

Unique to fermions

Well-defined **k**-states ("quasiparticles") with
- finite life time
- effective mass
- effective interaction

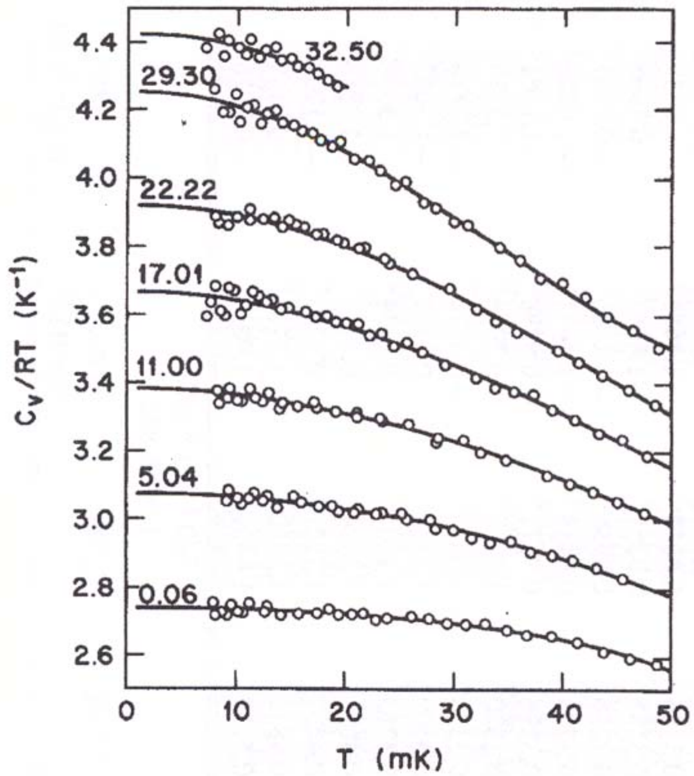
Universal Fermi liquid properties:

specific heat $$c_V = \frac{m^*}{m} c_V^0$$

spin susceptibility $$\chi_s = \frac{m^*/m}{1 + F_0^a} \chi_s^0$$

compressibility $$\kappa = \frac{m^*/m}{1 + F_0^s} \kappa^0$$

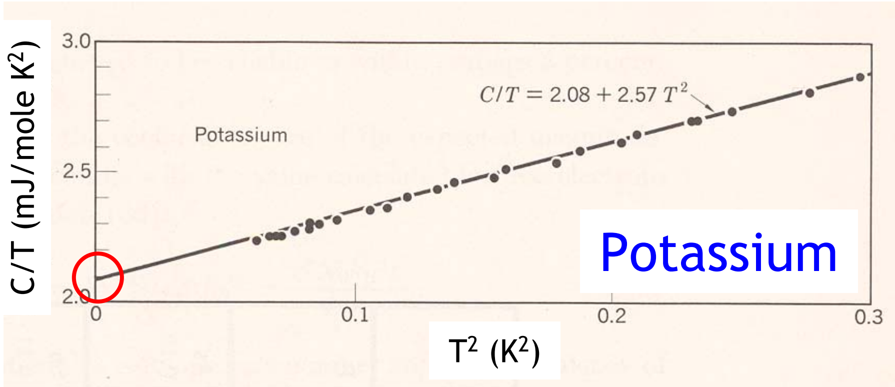
$$\lim_{T\to 0}\frac{c_V}{T}=\gamma\propto\frac{m^*}{m}$$ quasiparticle mass



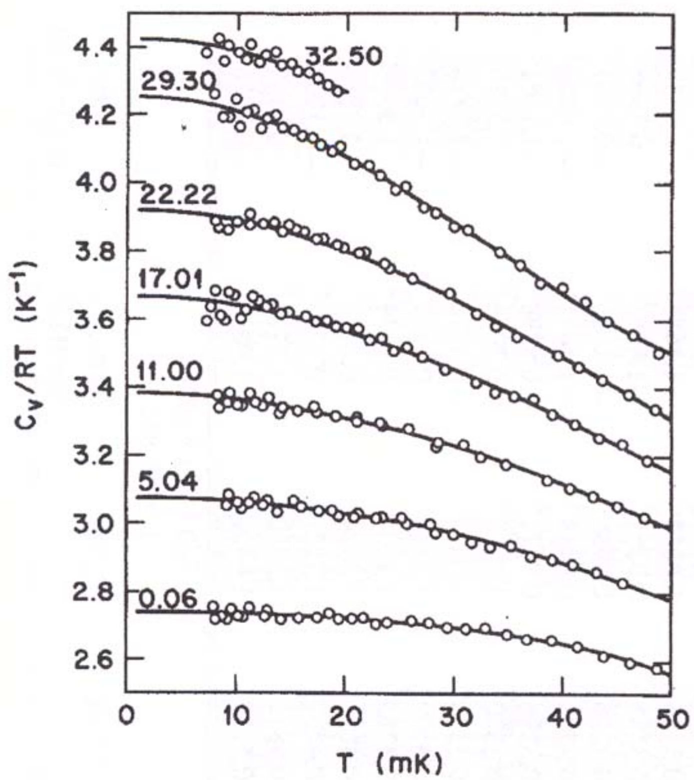
^3He

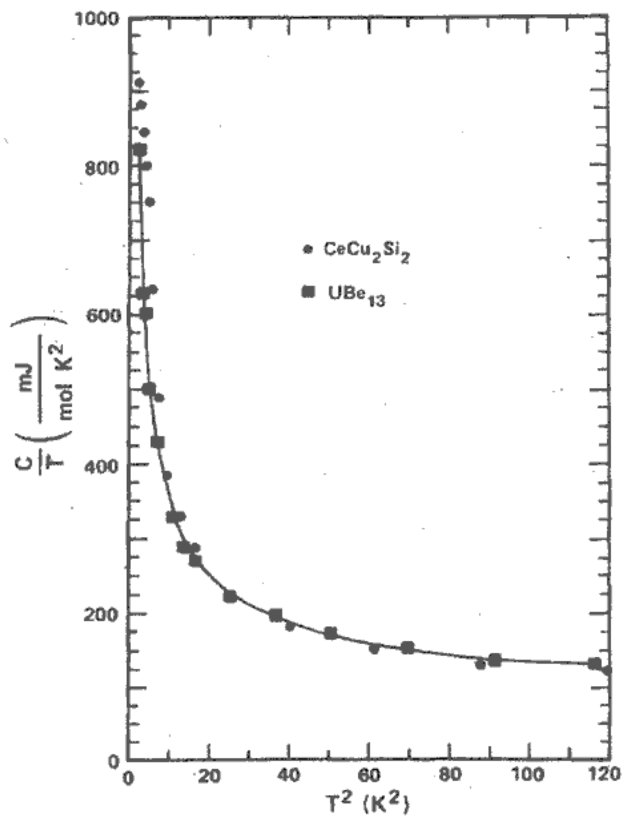
Greywall (1983)

Simple metals



$$\lim_{T \to 0} \frac{c_V}{T} = \gamma \propto \frac{m^*}{m}$$ quasiparticle mass



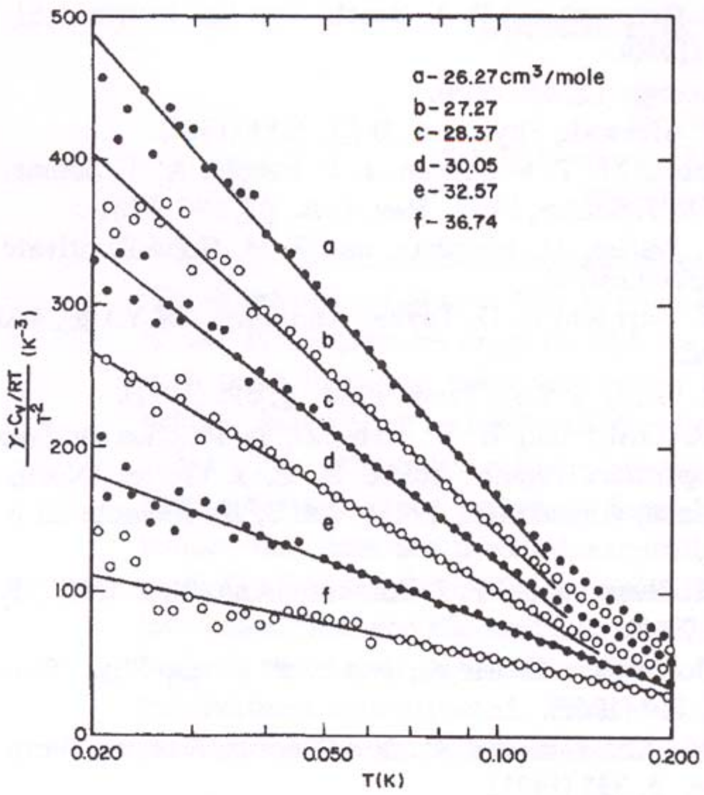


^{3}He
Greywall (1983)

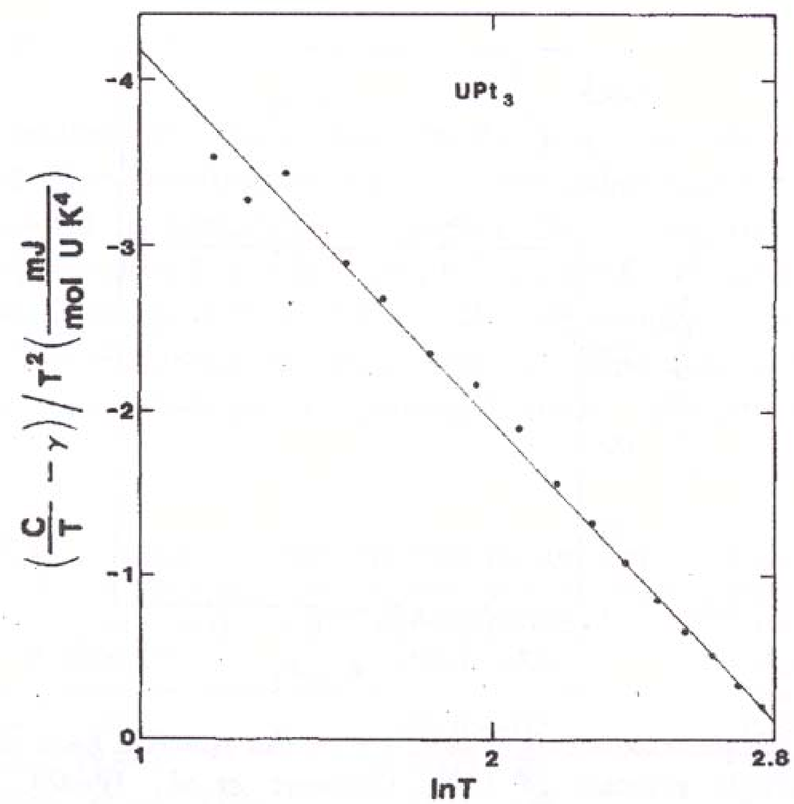
Heavy fermions: UBe_{13}, $CeCu_2Si_2$
Stewart *et al.* (1983,1984)

$$c_V = \gamma T + \Gamma T^3 \ln T$$

Eliashberg (1960)
Doniach, Engelsberg (1966)
Pethick, Carneiro (1973)
Chubukov *et al.* (2005)
Aleiner, Efetov (2006)

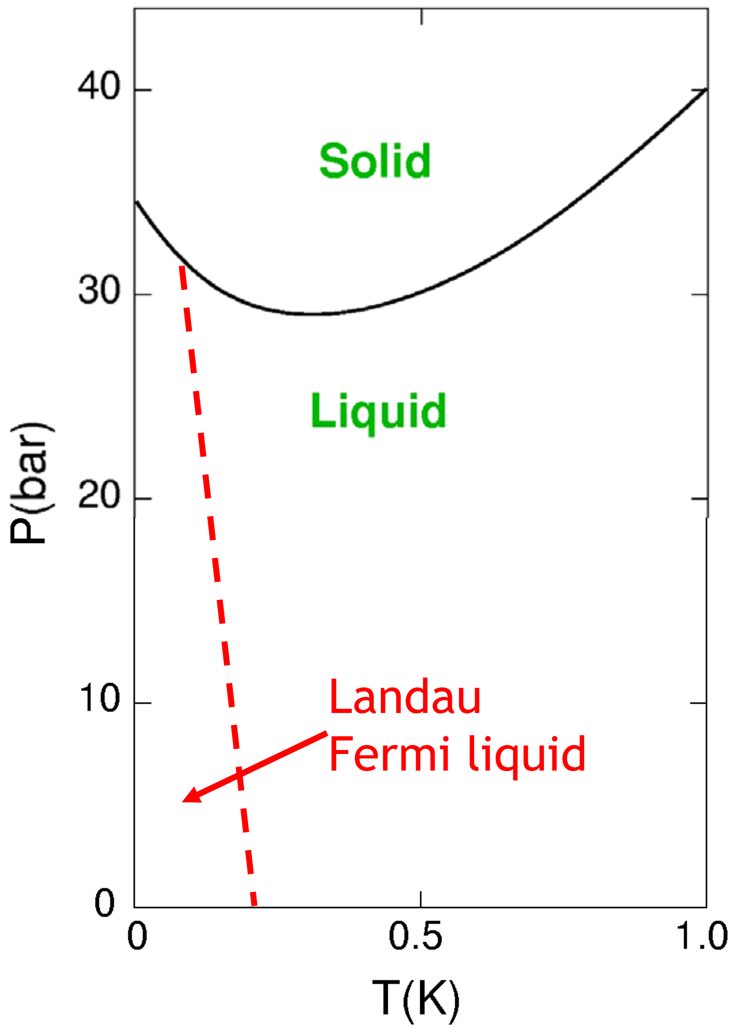


^{3}He
Greywall (1983)



Heavy fermions: UPt_3
Stewart *et al.* (1983,1984)

^3He

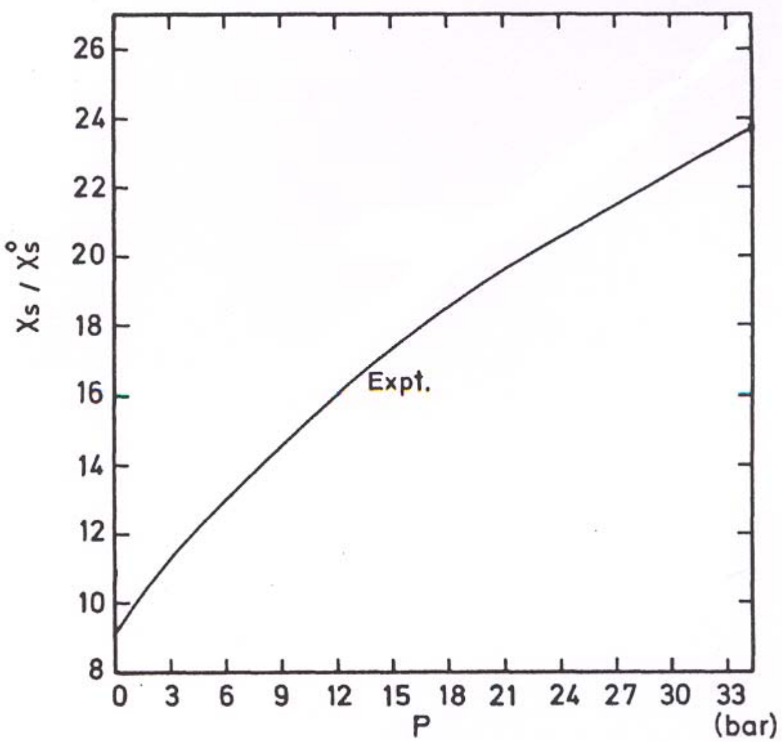


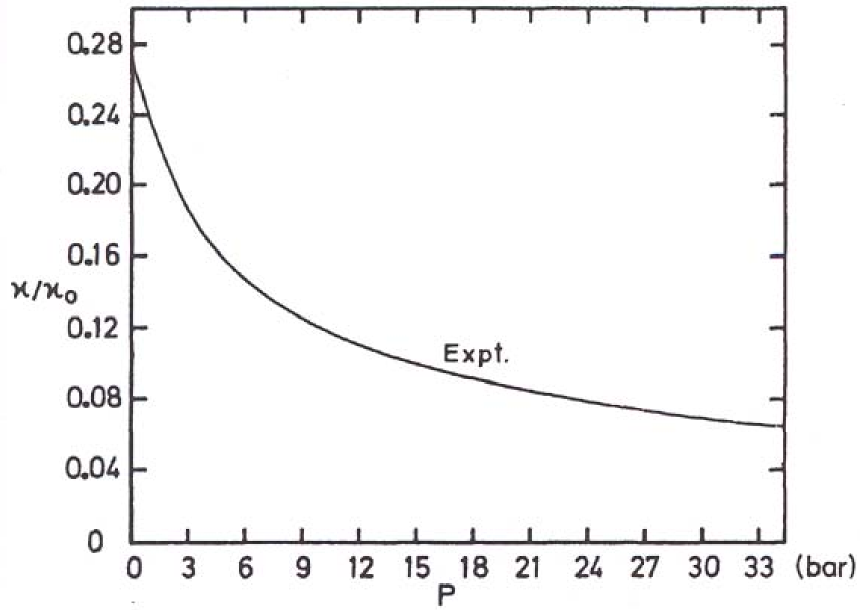
$$T_F^* = \frac{1}{k_B}\frac{\hbar^2 k_F^2}{2m^*}$$

$$= \frac{m}{m^*} T_F \approx 1.5K$$

^3He: Pressure dependence

Greywall (1983)



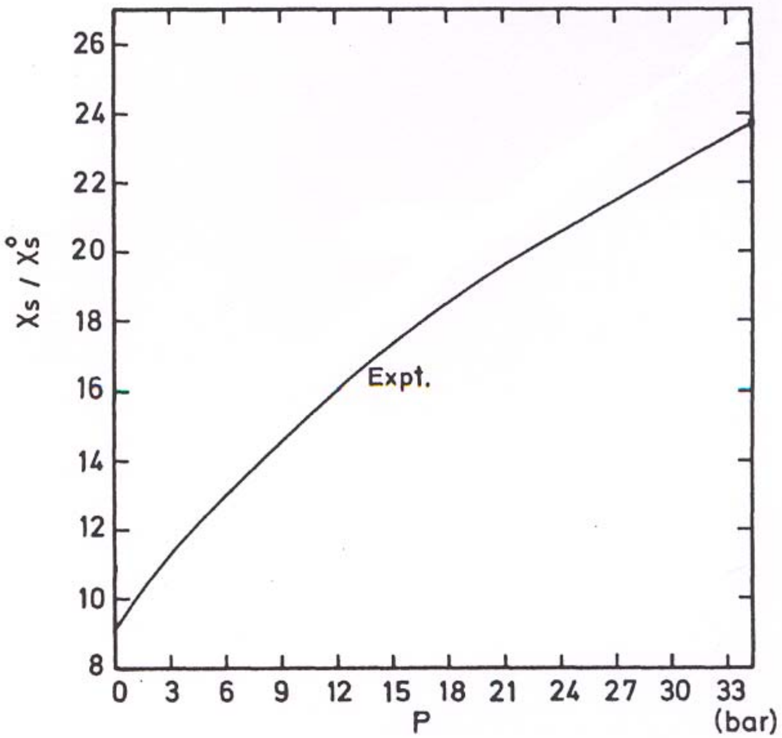


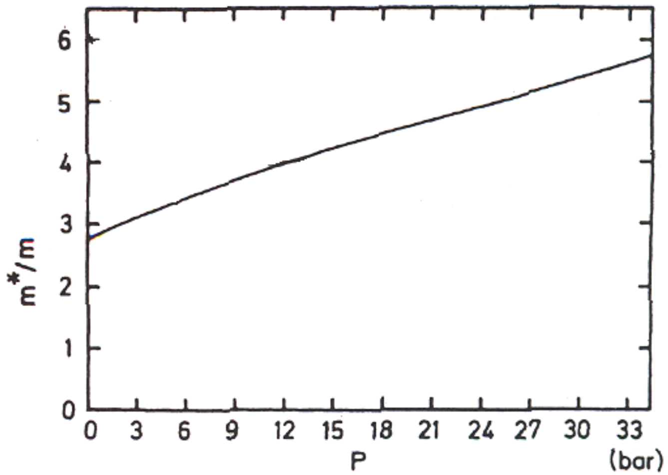
Strong short-range, repulsive interactions →

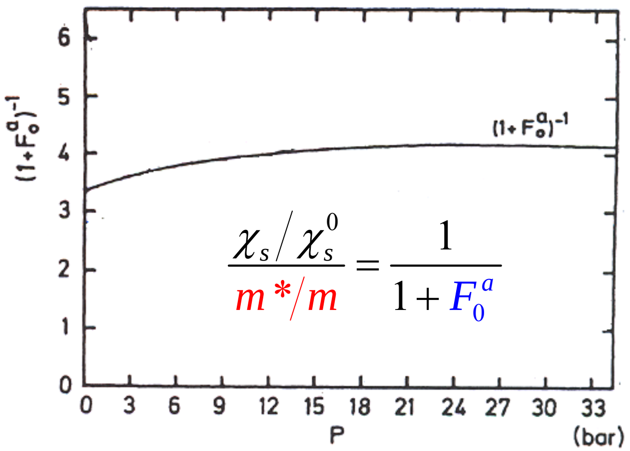
- spin fluctuations strongly enhanced
- density fluctuations strongly suppressed

High susceptibility: "almost ferromagnetic liquid"?

^{3}He: Pressure dependence







$$\frac{\chi_s/\chi_s^0}{m^*/m} = \frac{1}{1+F_0^a}$$

$$\chi_s/\chi_s^0 = \frac{m^*/m}{1+F_0^a}$$

Anderson, Brinkman (1975):

"Almost localized Fermi liquid";
vicinity of Mott transition

Gutzwiller-Brinkman-Rice theory

$$H = \sum_{i,j,\sigma} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + U \underbrace{\sum_{i} n_{i\uparrow} n_{i\downarrow}}_{\hat{D}}$$

Gutzwiller (1963)
Hubbard (1963)
Kanamori (1963)

Gutzwiller wave function $|\psi_G\rangle = e^{-\lambda \hat{D}} |\psi_0\rangle$

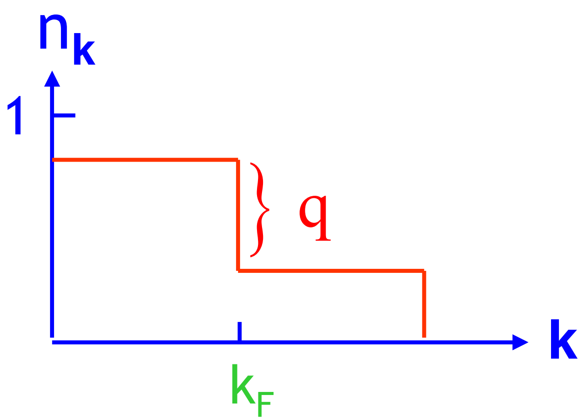
One-particle wave function (Hartree-Fock, BCS, etc.)

$$E_G = \frac{1}{L} \frac{\langle \psi_G | H | \psi_G \rangle}{\langle \psi_G | \psi_G \rangle}$$

d=1,∞: exact analytic evaluation possible   Metzner, DV (1988/89)

Gutzwiller approximation (1963/65)

$$\frac{E_G^{GA}}{L} = q(d)\bar{\varepsilon}_0 + Ud, \qquad \frac{\partial E}{\partial d} = 0$$



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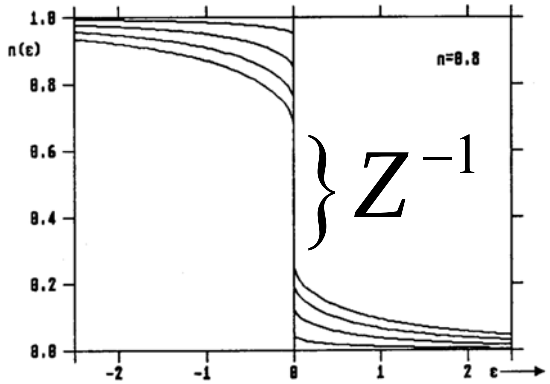


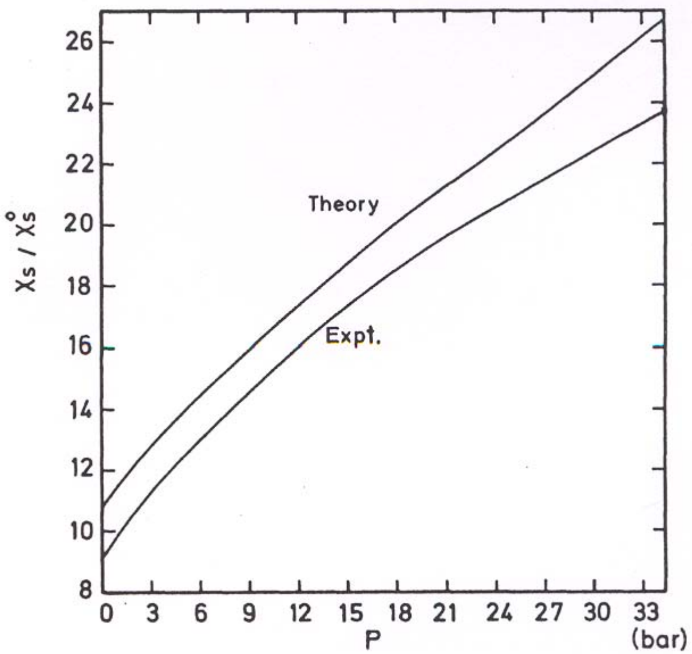
Brinkman, Rice (1970): $\frac{m^*}{m} = q^{-1} \xrightarrow{U \to U_c} \infty$

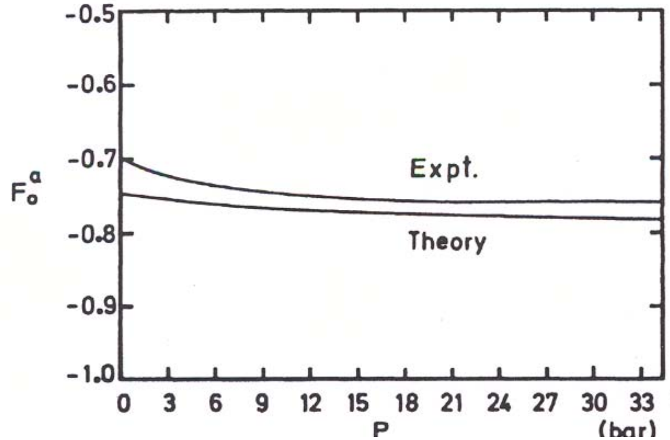
Describes metal-insulator ("Mott") transition,
→ application to V_2O_3

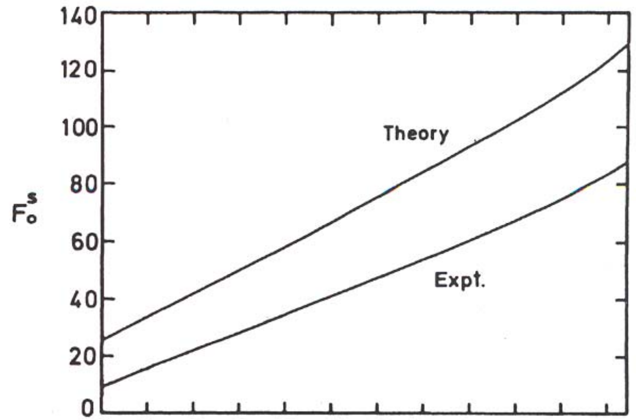
Gutzwiller-Brinkman-Rice theory:
Application to normal liquid ^{3}He

DV (1984)

Gutzwiller approximation ⇔
Landau Fermi liquid theory







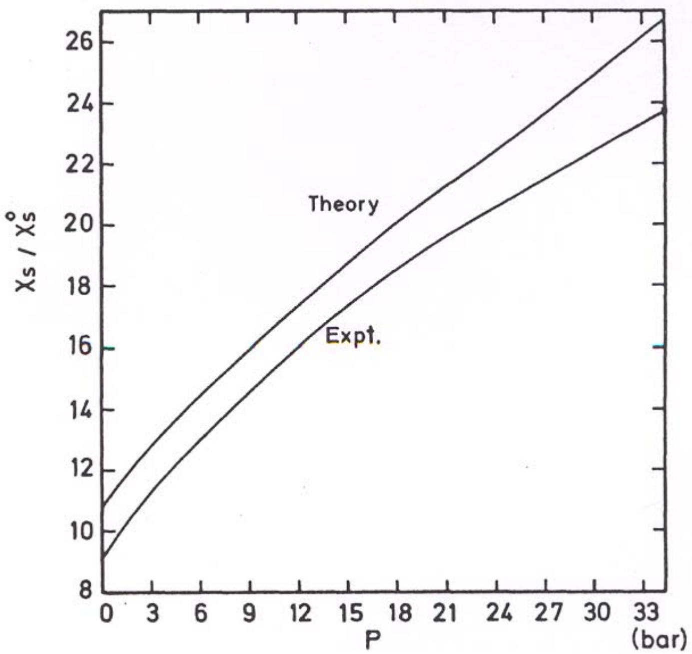
liquid ^{3}He:

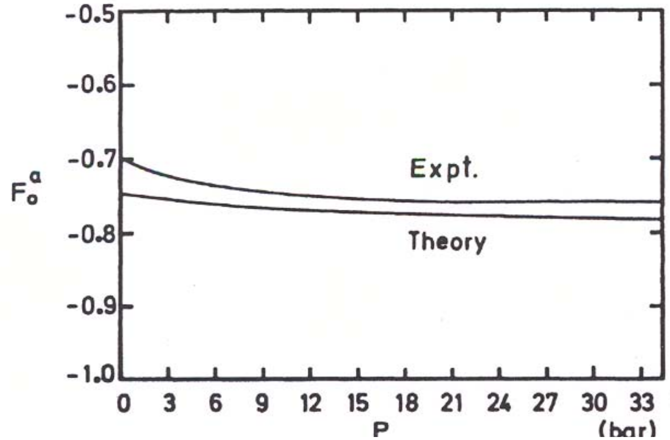
"almost localized Fermi liquid" (vicinity of Mott transition), not „almost ferromagnetic"

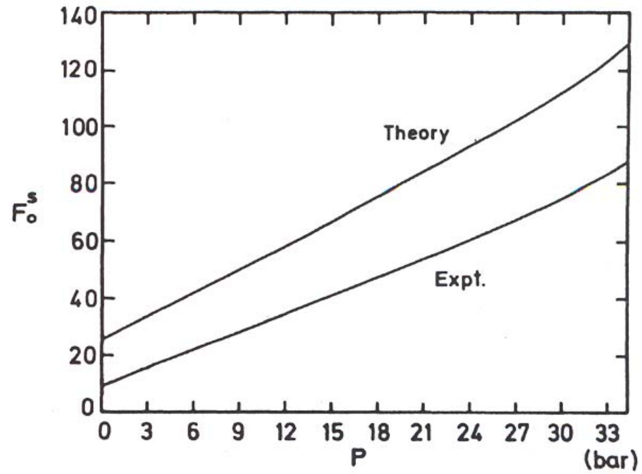
Gutzwiller-Brinkman-Rice theory:
Application to normal liquid ^{3}He

DV (1984)

Gutzwiller approximation ⇔
Landau Fermi liquid theory







Generalization:
Gutzwiller-Hubbard
lattice gas model for ^{3}He

DV, Wölfle, Anderson (1987)

Gutzwiller wave function in d→∞

$$\left|\psi_G\right\rangle = e^{-\lambda \hat{H}_U}\left|\psi_0\right\rangle$$

$$E_G = \frac{\left\langle\psi_G\right|H\left|\psi_G\right\rangle}{\left\langle\psi_G\middle|\psi_G\right\rangle}$$

Gutzwiller approximation exact in d→∞

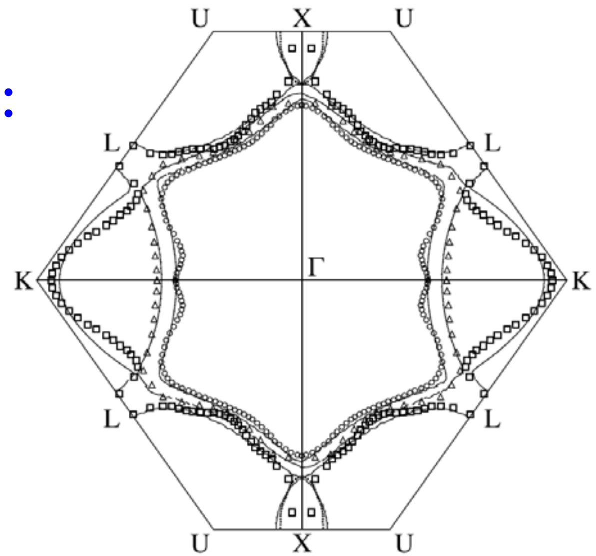
Metzner, DV (1989)

d→∞: Evaluation of E_G for arbitrary $\left|\psi_0\right\rangle$

Gebhard (1990)

Multi-band generalization:

„Gutzwiller DFT“



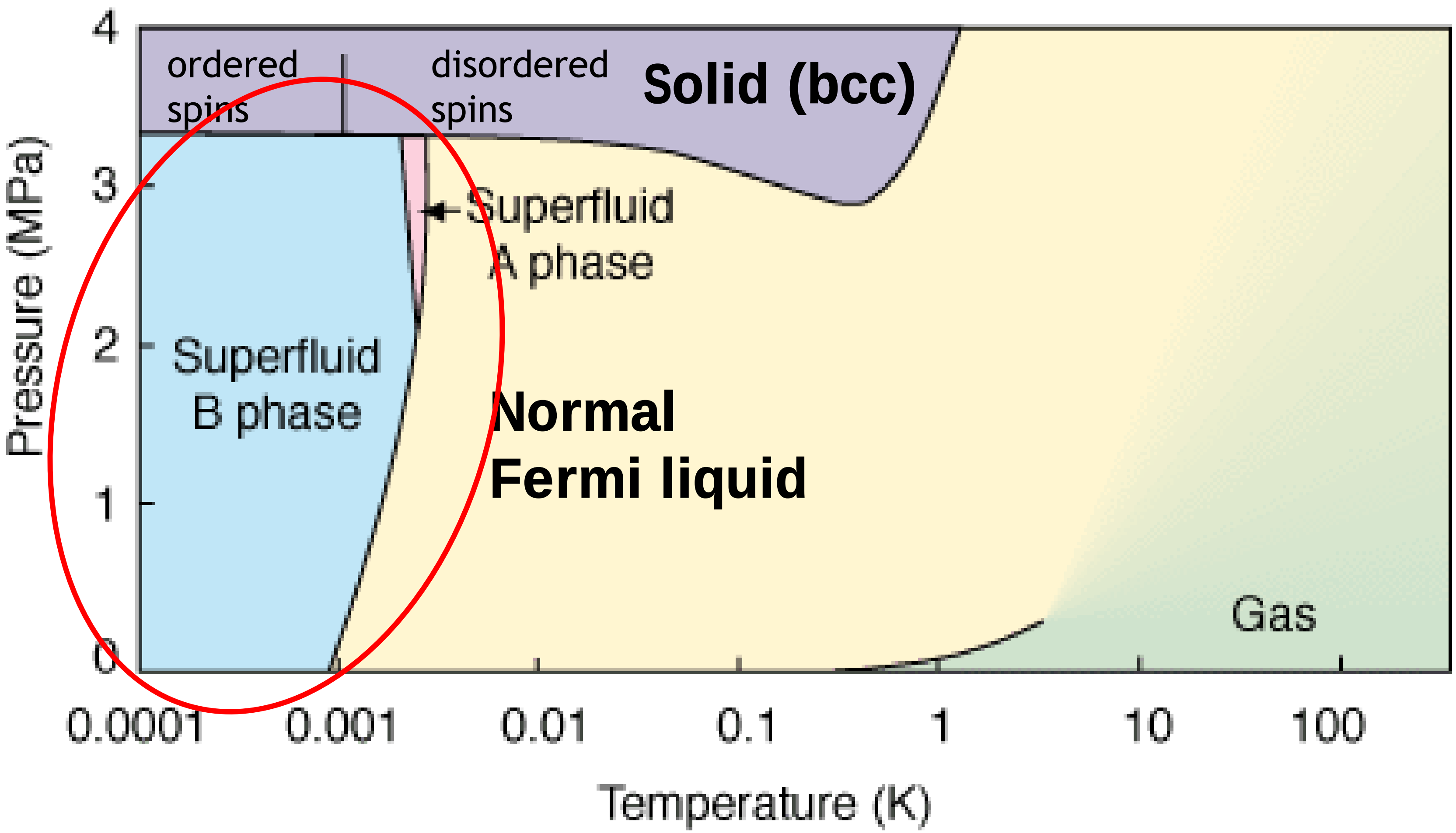
Ferromagnetic Ni:
Cut of Fermi surface

Bünemann, Gebhard, Ohm, Weiser, Weber (2005)

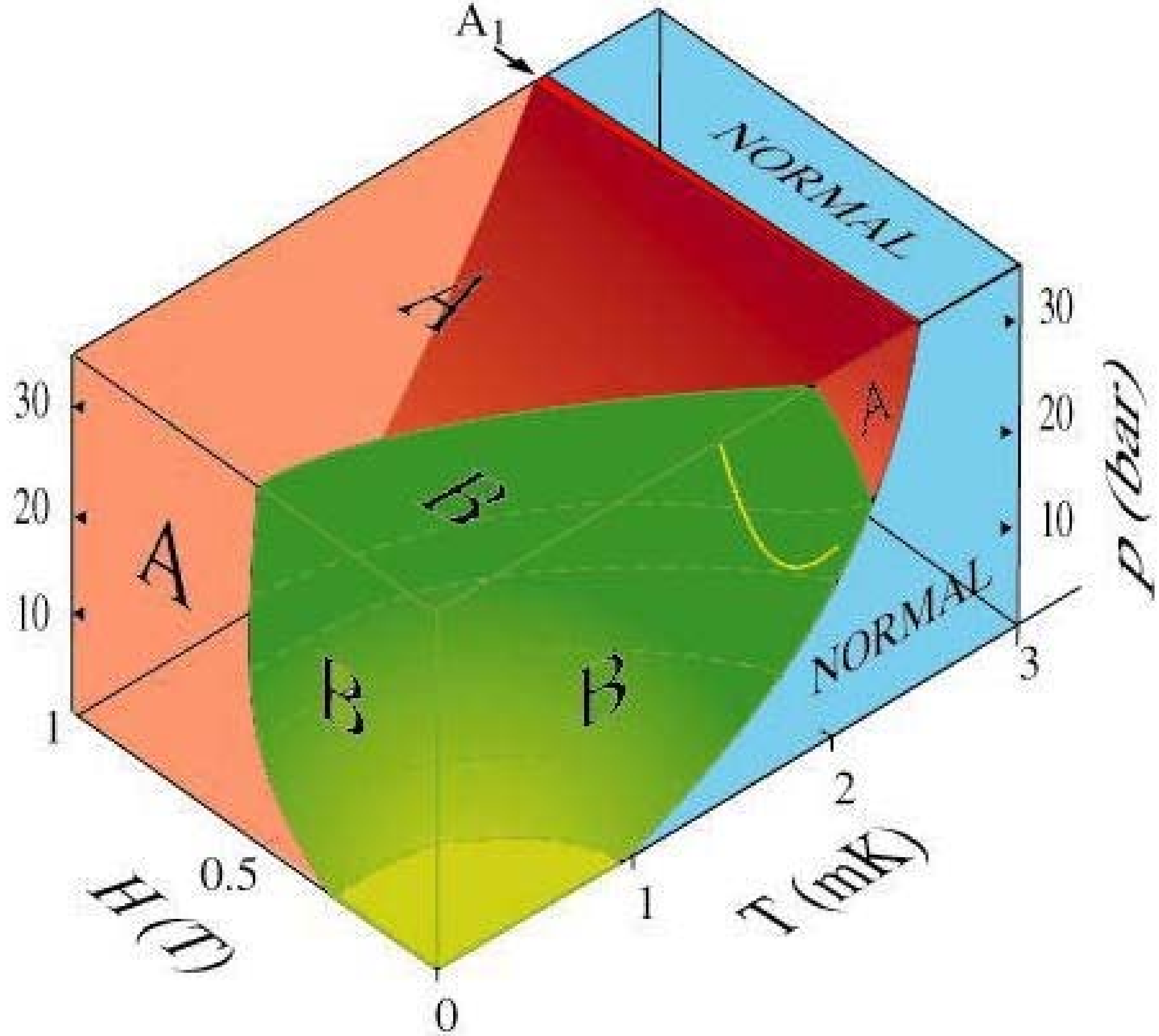
2. Pair-correlated state

Osheroff, Richardon, Lee (1972)

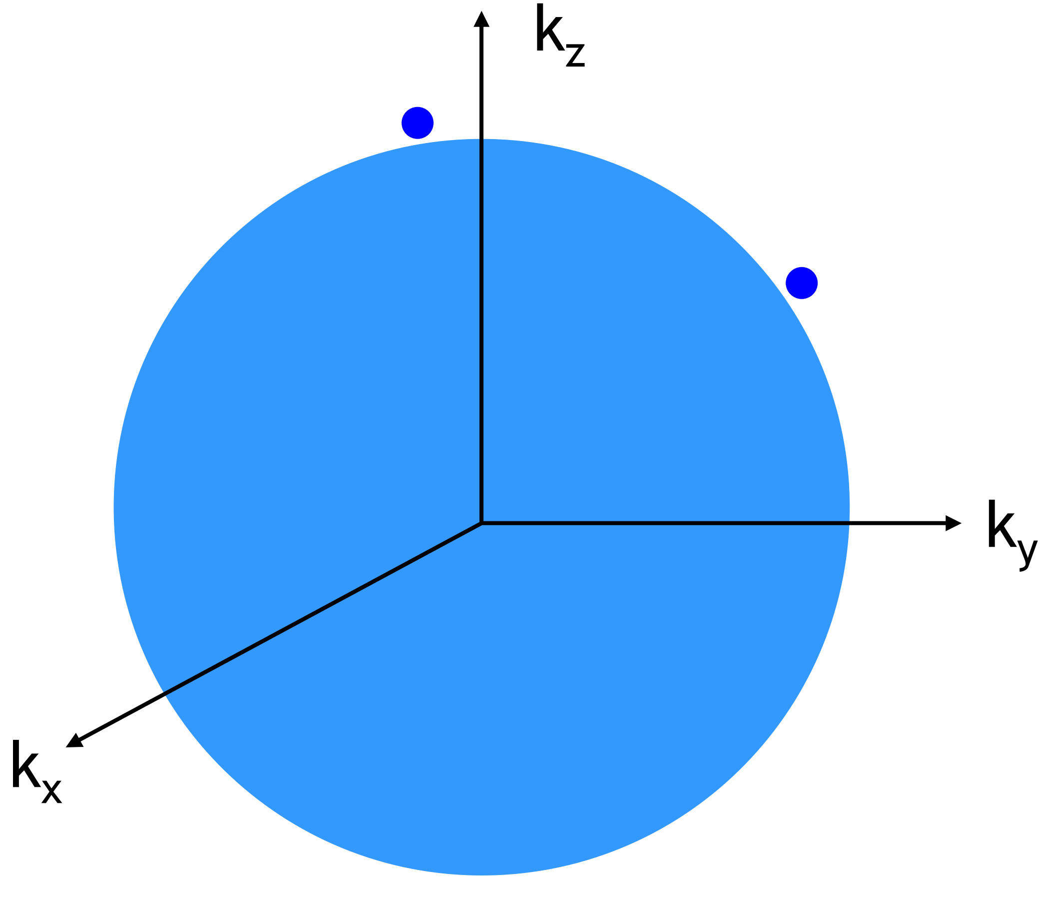
^{3}He: P-T phase diagram



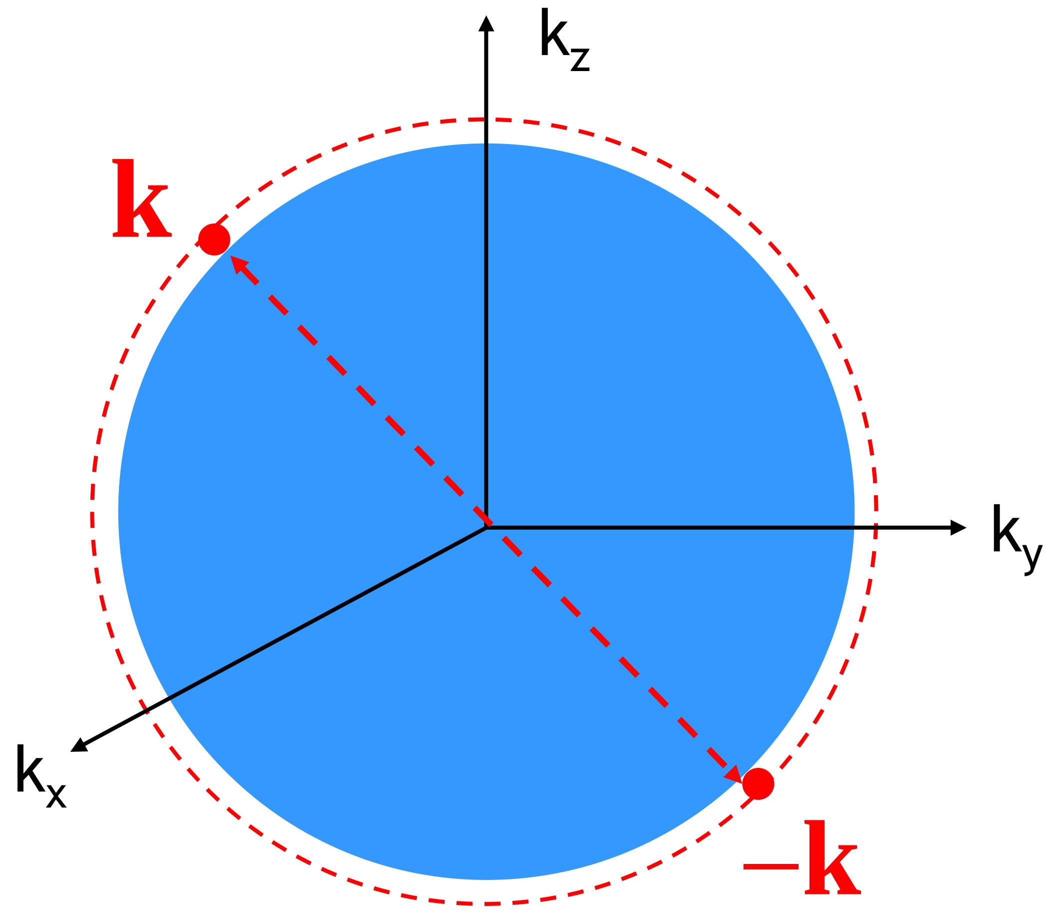
^{3}He: H-P-T phase diagram



Landau Fermi liquid



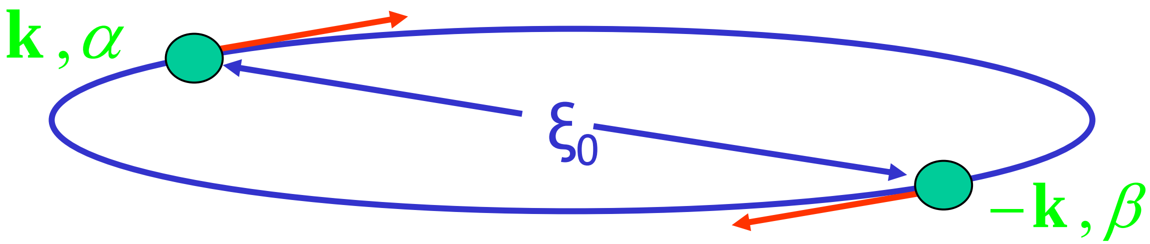
Arbitrarily weak attraction $\Longrightarrow$ Cooper instability



Universal fermionic property

Arbitrarily weak attraction:

Cooper pair $(\mathbf{k},\alpha;-\mathbf{k},\beta)$



$$\Psi_{L=0,2,4,\ldots} = \psi(\mathbf{r})\left|\uparrow\downarrow - \downarrow\uparrow\right\rangle$$

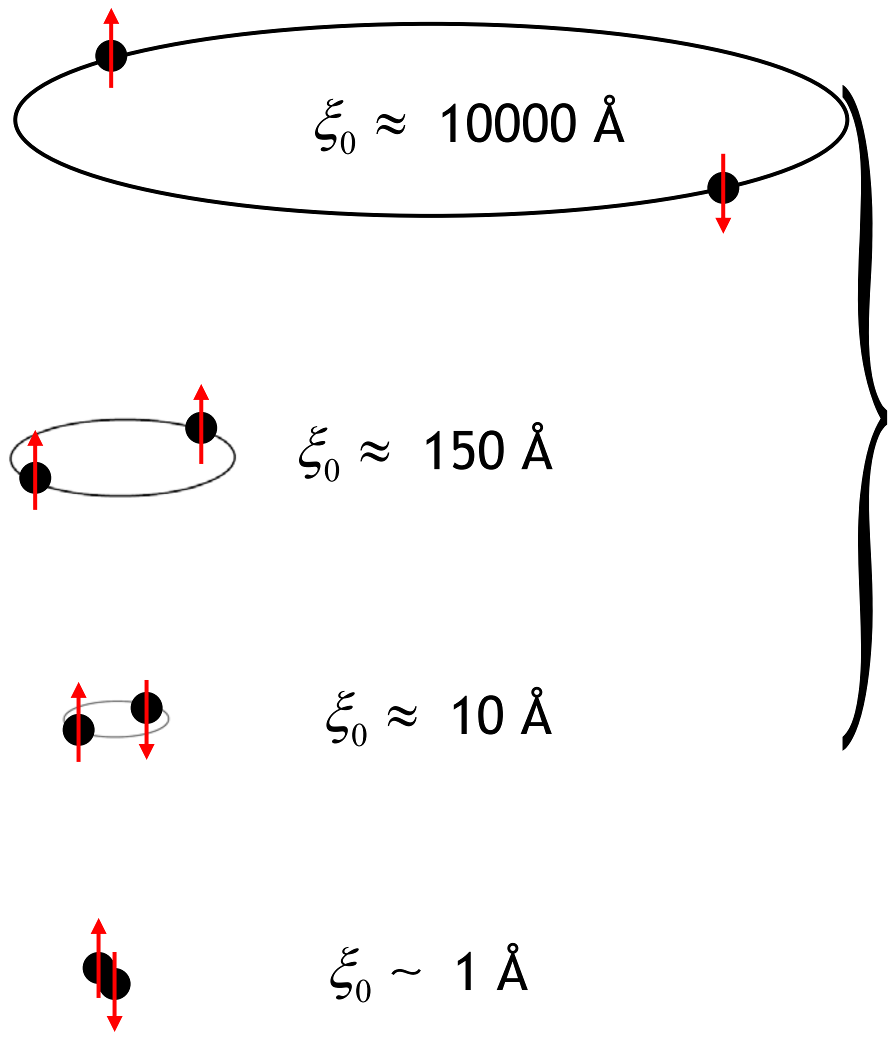
S=0
(singlet)

$$\Psi_{L=1,3,5,\ldots} = \psi_+(\mathbf{r})\left|\uparrow\uparrow\right\rangle + \psi_0(\mathbf{r})\left|\uparrow\downarrow + \downarrow\uparrow\right\rangle + \psi_-(\mathbf{r})\left|\downarrow\downarrow\right\rangle$$

S=1
(triplet)

Cooper pairing of Fermions vs. Bose-Einstein condensation

Cooper pair: “Quasi-boson“



Conventional superconductors — $\xi_0 \approx$ 10000 Å

Superfluid ^{3}He — $\xi_0 \approx$ 150 Å

High T_C superconductors — $\xi_0 \approx$ 10 Å

Tightly packed bosons — $\xi_0 \sim$ 1 Å

BCS

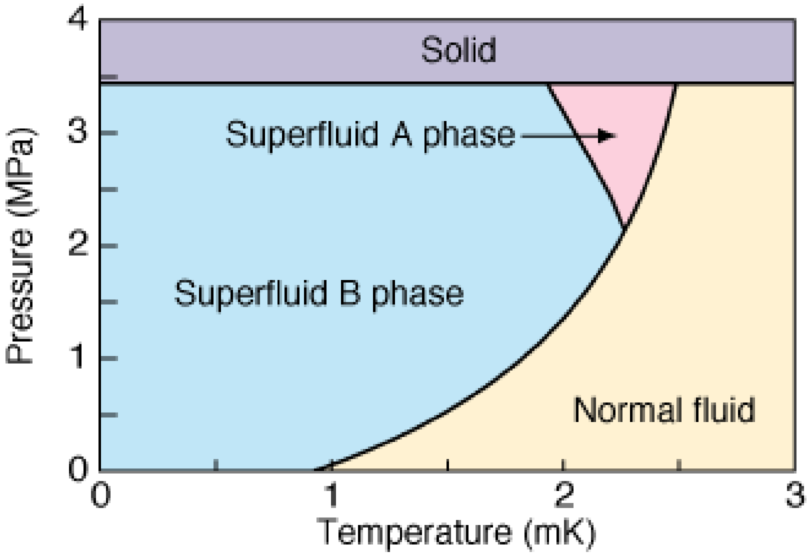
Continuous crossover?

BEC

New insights from BEC of cold atoms

Leggett (1980)

Superfluid ^3He



L=1, S=1

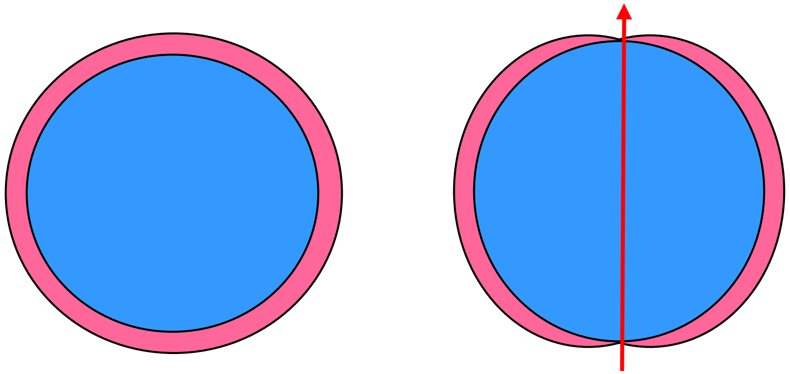
Order parameter matrix

$A_{i\mu}$

$3 \times 3 \times 2 = 18$ real numbers

B phase

A phase



$SO(3)_S \times SO(3)_L \times U(1)_\varphi$ symmetry broken

Leggett (1975)
Mineev (1980)
Bruder, DV (1986)

s-wave pairing

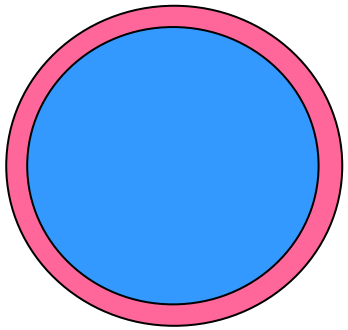
"conventional" superfluidity/superconductivity

$SO(3)_S \times SO(3)_L \times U(1)_\varphi$ symmetry broken

Leggett (1975)
Mineev (1980)
Bruder, DV (1986)

$SO(3)_{S+L}$ —

B phase



Spontaneously broken spin-orbit symmetry (SBSOS) Leggett (1972)

"unconventional" superfluidity

$$c_V \propto e^{-\Delta / k_B T}$$

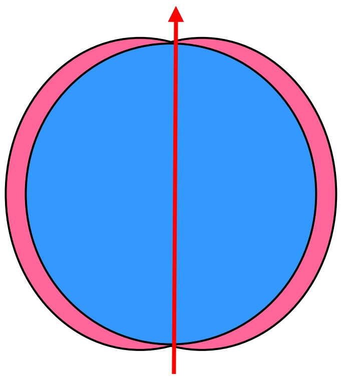
only stable phase in mean-field theory

$SO(3)_S \times SO(3)_L \times U(1)_\varphi$ symmetry broken

$U(1)_{S_z} \times U(1)_{L_z - \varphi}$

Leggett (1975)
Mineev (1980)
Bruder, DV (1986)

A phase



$$c_V \propto T^3$$

"unconventional" superfluidity

stabilization by
strong-coupling effects

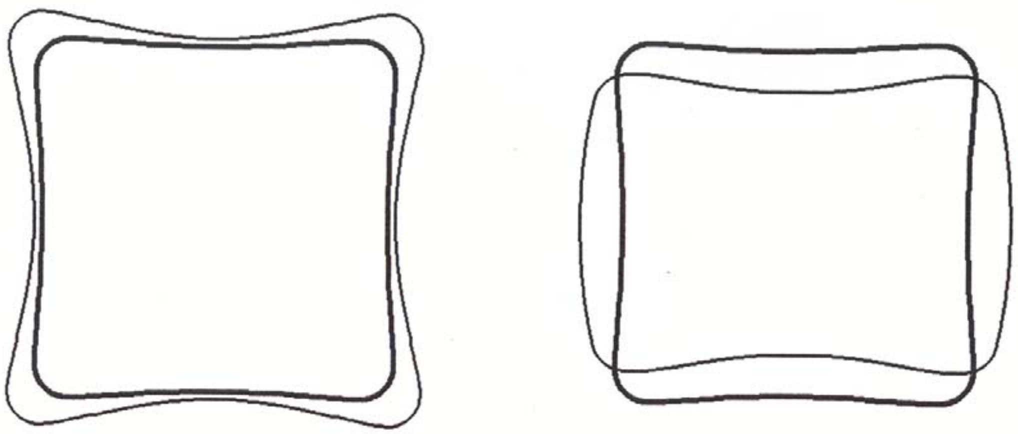
Unconventional superconductivity

$G \times SO(3)_S \times T \times U(1)_\varphi$ symmetry broken

G: point symmetry group of solid

Anderson (1985)
Volovik, Gorkov (1985)
Ueda, Rice (1985)
Blount (1985)

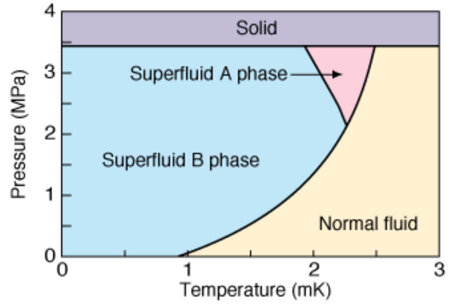
Example: Tetragonal crystal (D_{4h})



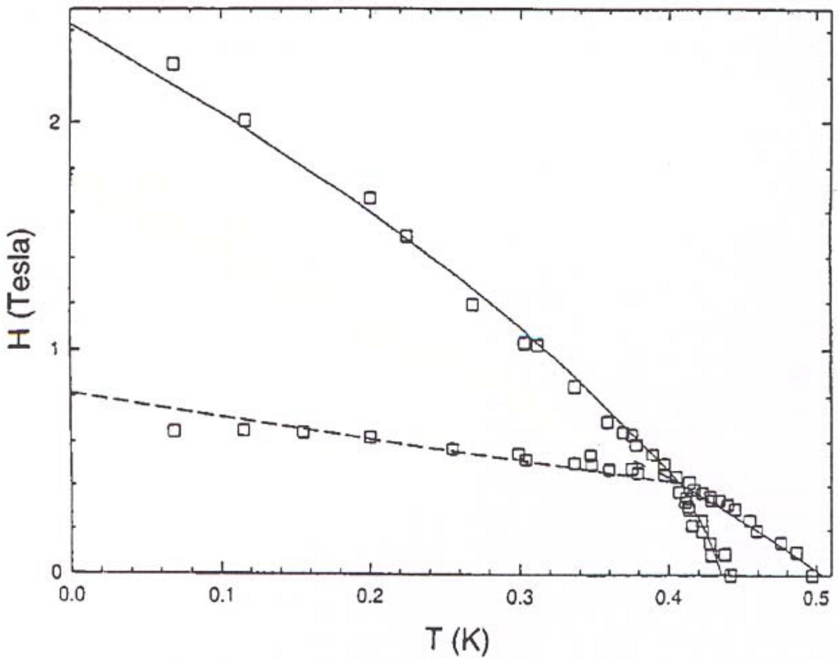
conventional

D_{2h}: unconventional

Unconventional superconductivity



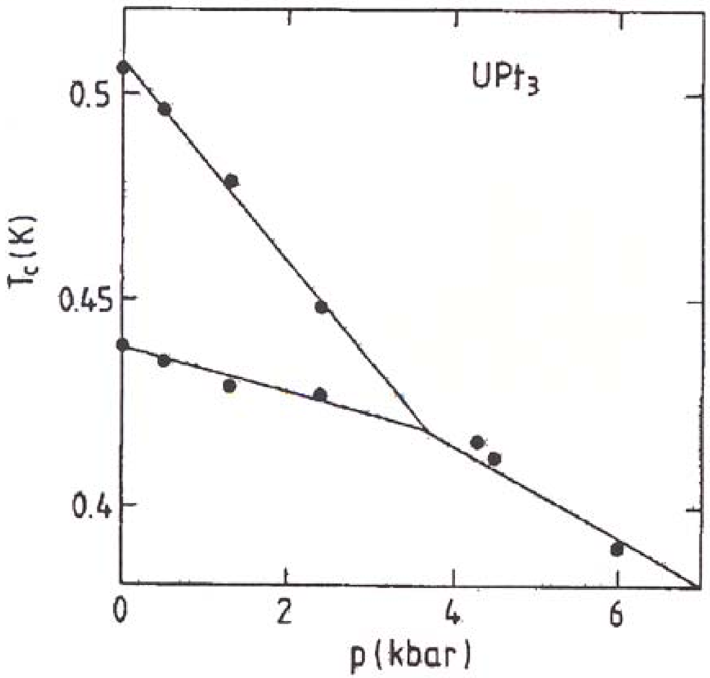
UPt_3



H-T phase diagram

Adenwalla *et al.* (1990)

Park, Joynt (1995)



T-P phase diagram

v. Löhneysen, Trappmann, Taillefer (1992)

Unconventional superconductivity

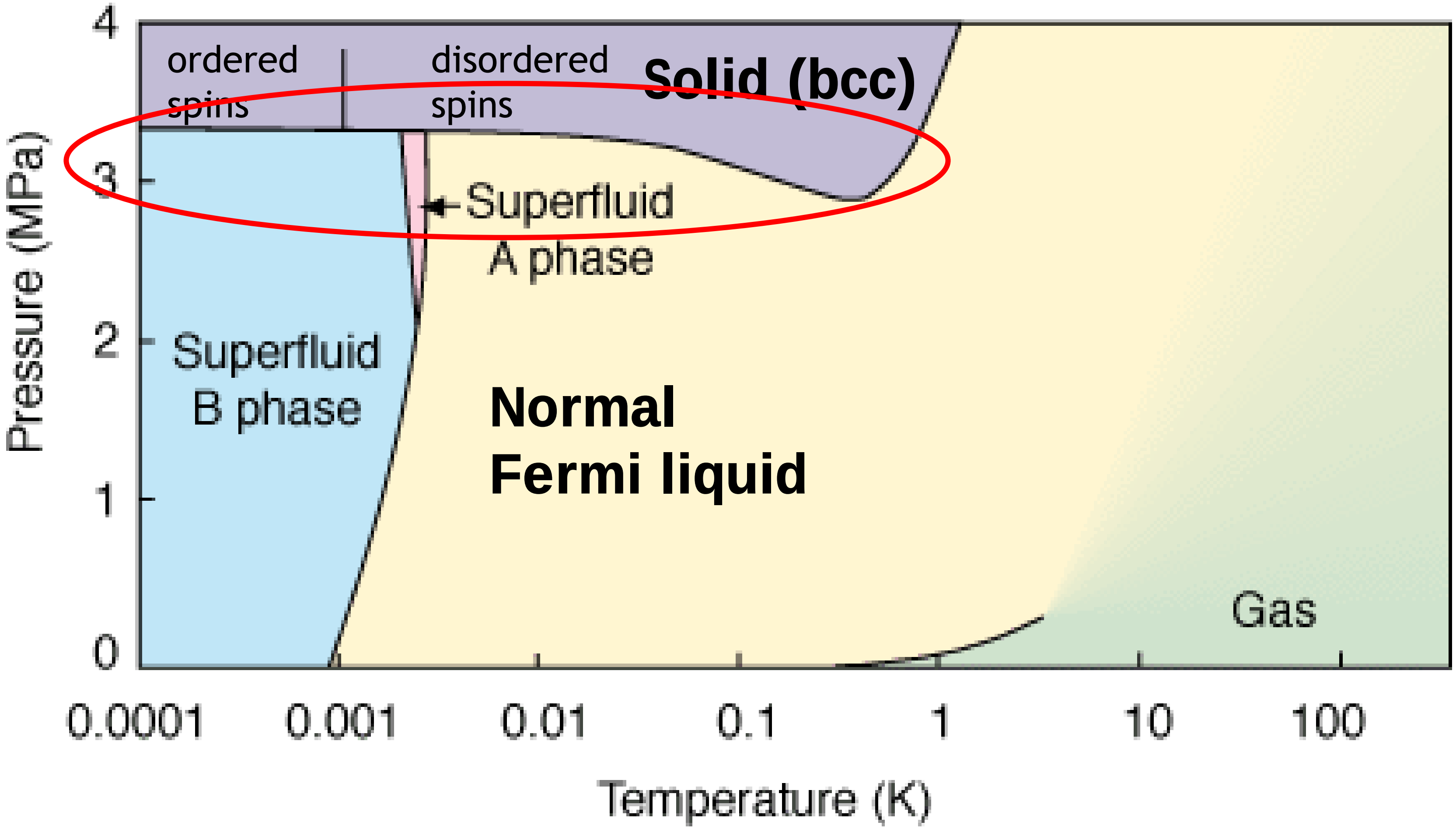
Other systems:

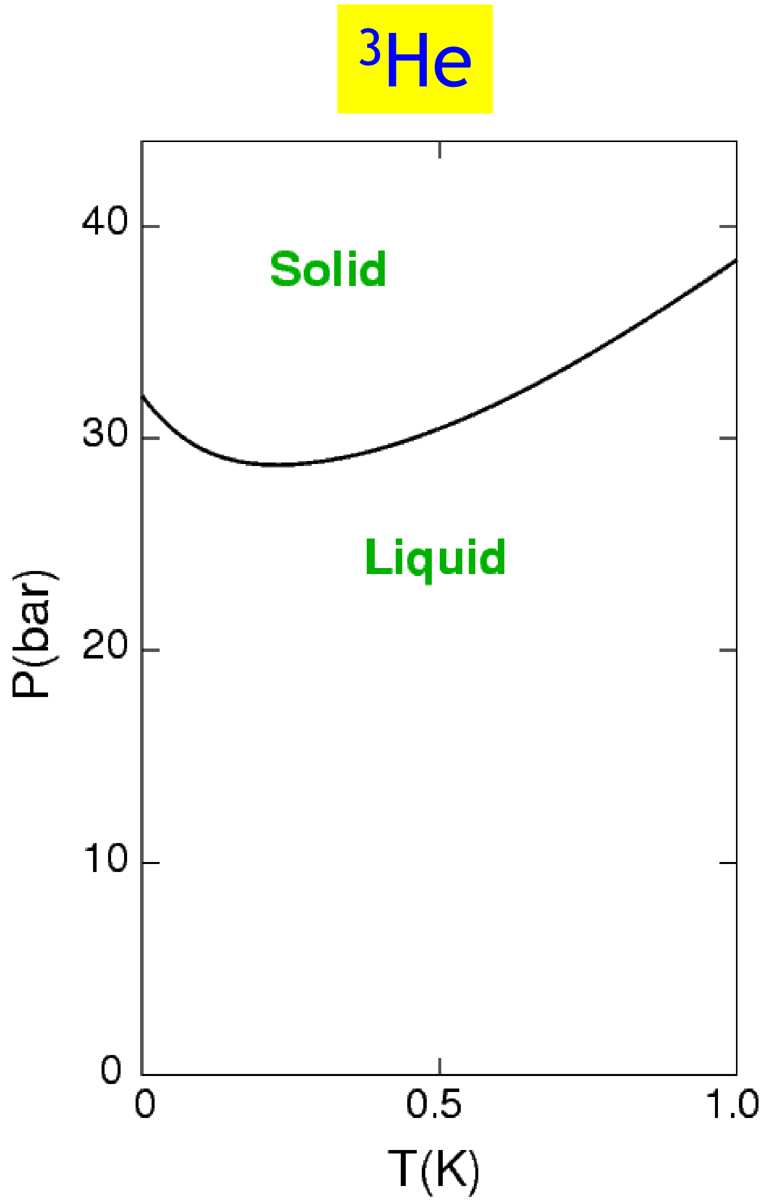
$U_{1-x}Th_xBe_{13}$ Ott *et al.* (1985)

Sr_2RuO_4 Maeno *et al.* (1994) → triplet pairing

3. Localization-delocalization transition

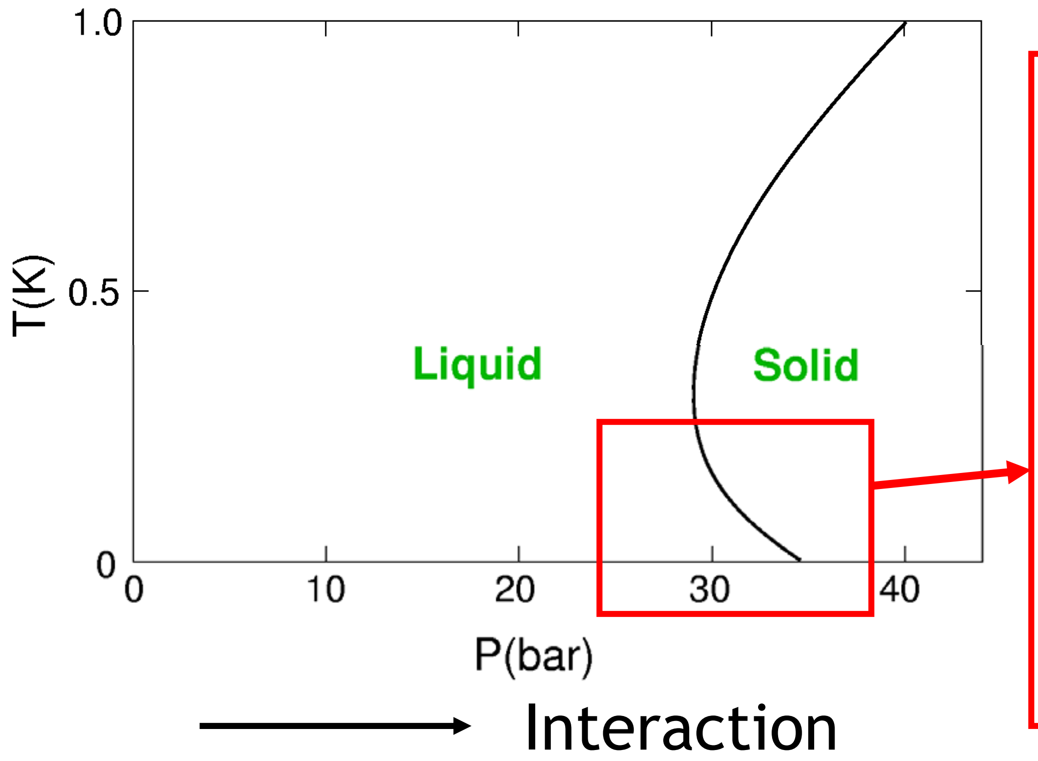
^{3}He

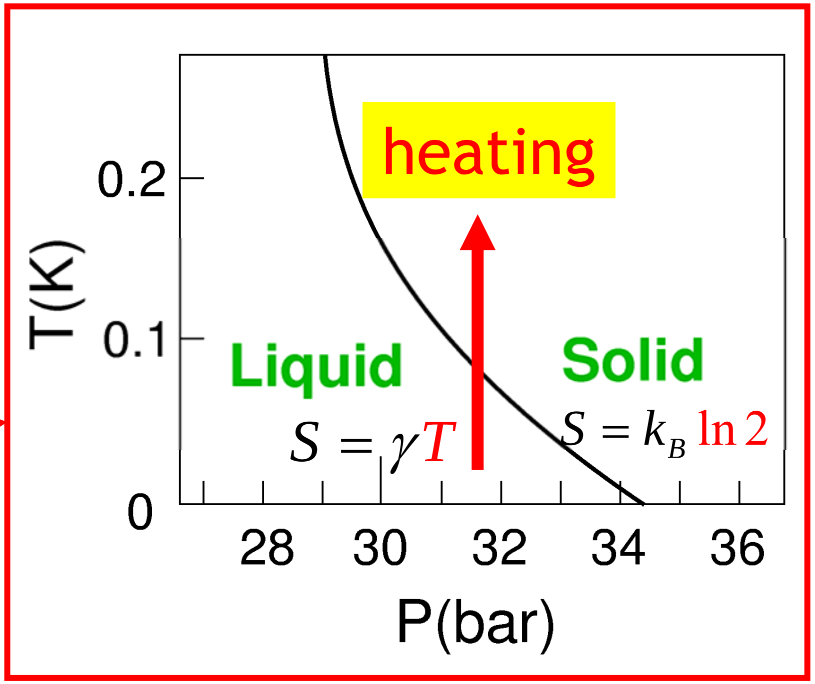




^{3}He
Solid
Liquid
P(bar)
T(K)
0
10
20
30
40
0
0.5
1.0

^{3}He





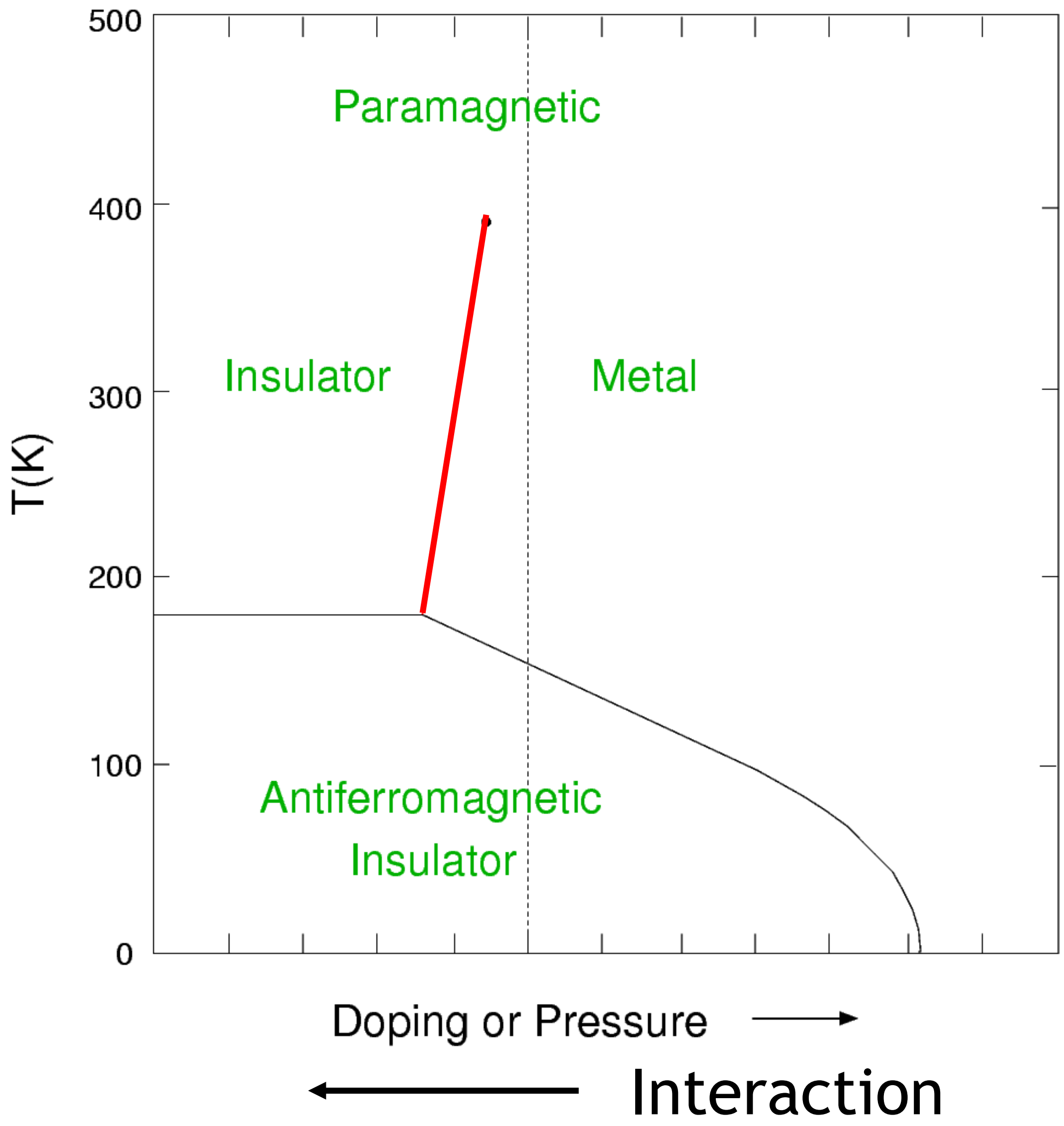
Clausius-Clapeyron eq.:

$$\frac{dP}{dT} = \frac{s_2 - s_1}{v_2 - v_1} = \frac{\Delta s}{\Delta v}.$$

anomalous ("Pomeranchuk") effect

V_2O_3

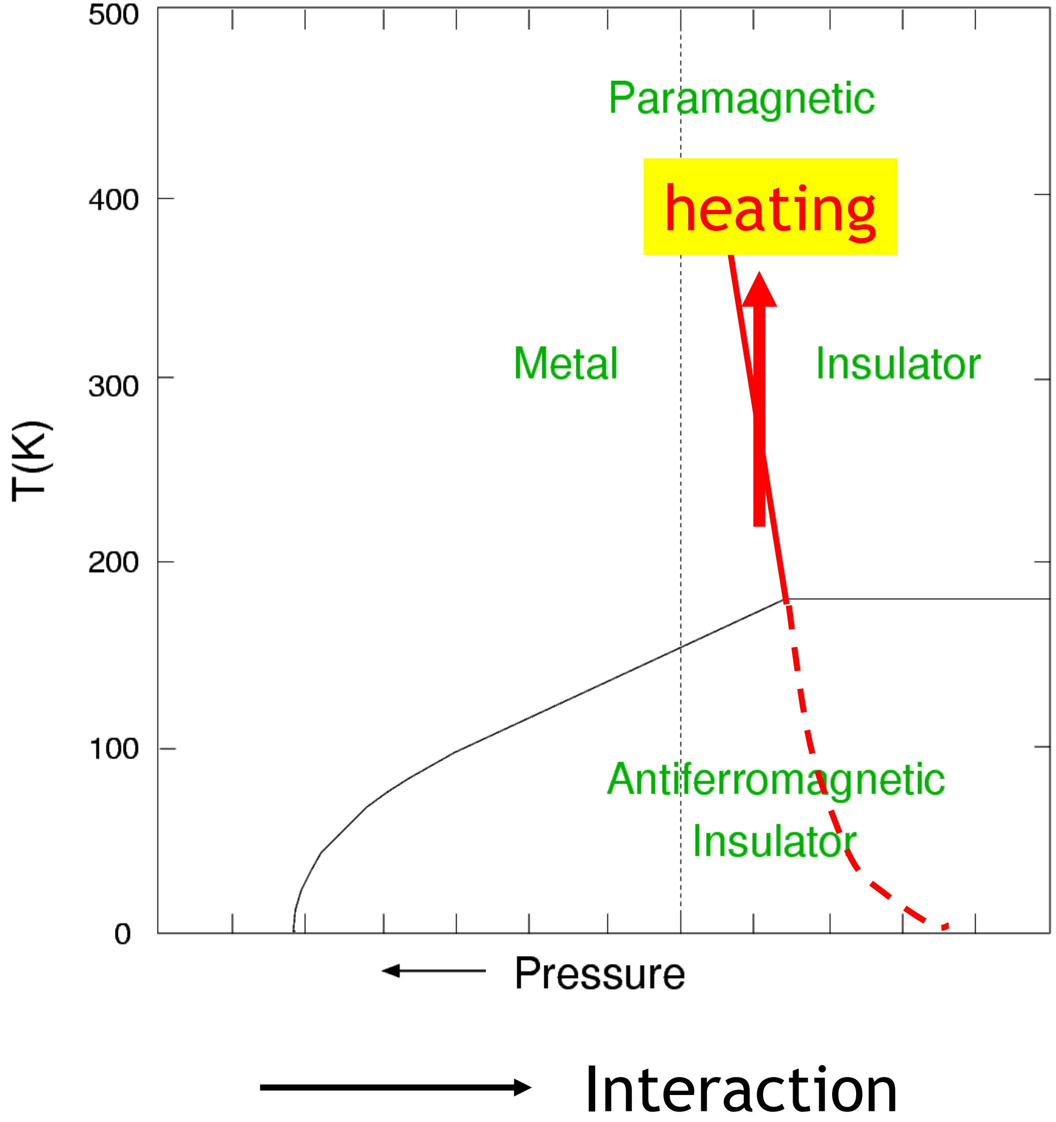
McWhan *et al.* (1971)



Correlation induced ("Mott-Hubbard") metal-insulator transition

V_2O_3

McWhan *et al.* (1971)



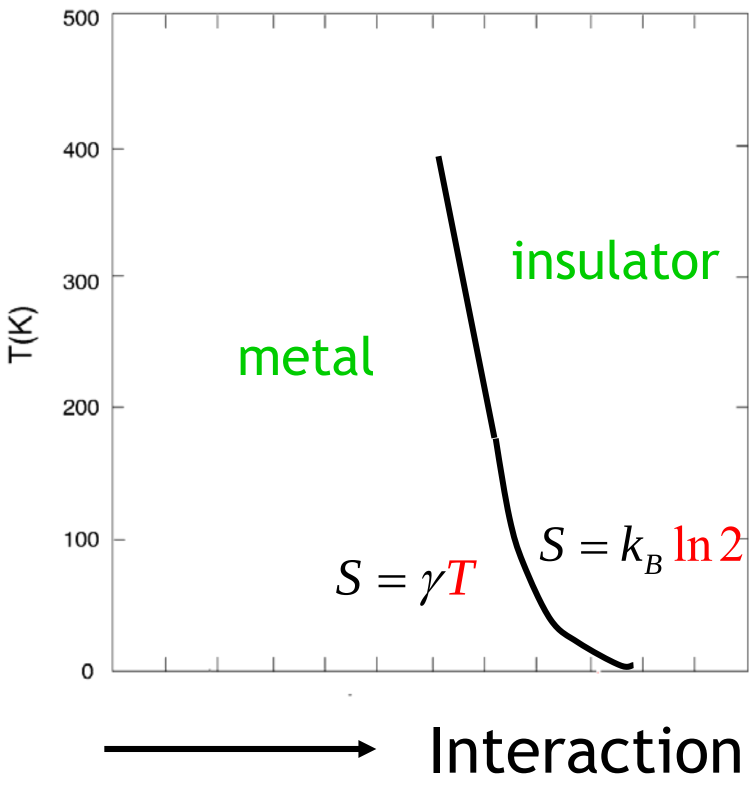
Clausius-Clapeyron eq.:

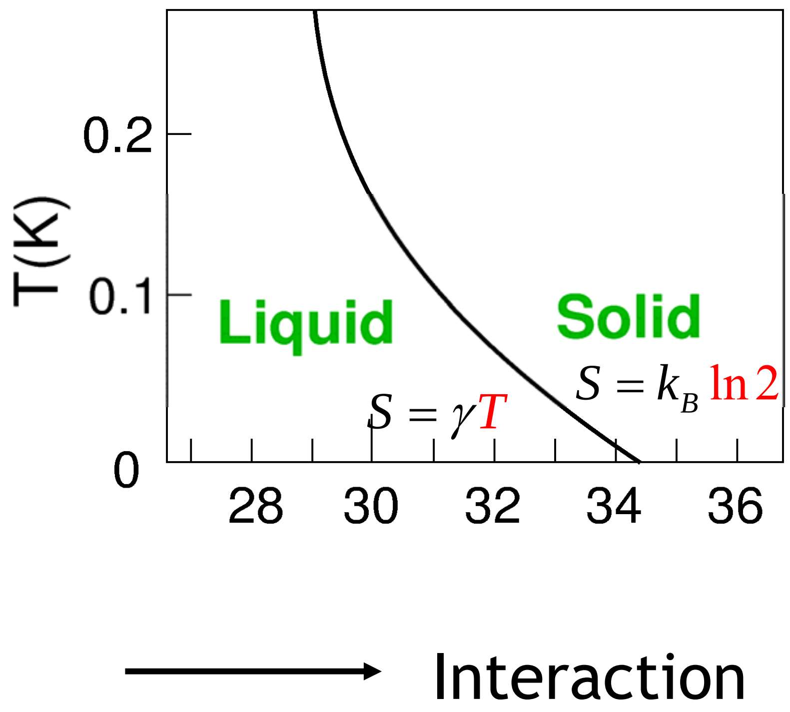
$$\frac{dP}{dT} = \frac{s_2 - s_1}{v_2 - v_1} = \frac{\Delta s}{\Delta v}.$$

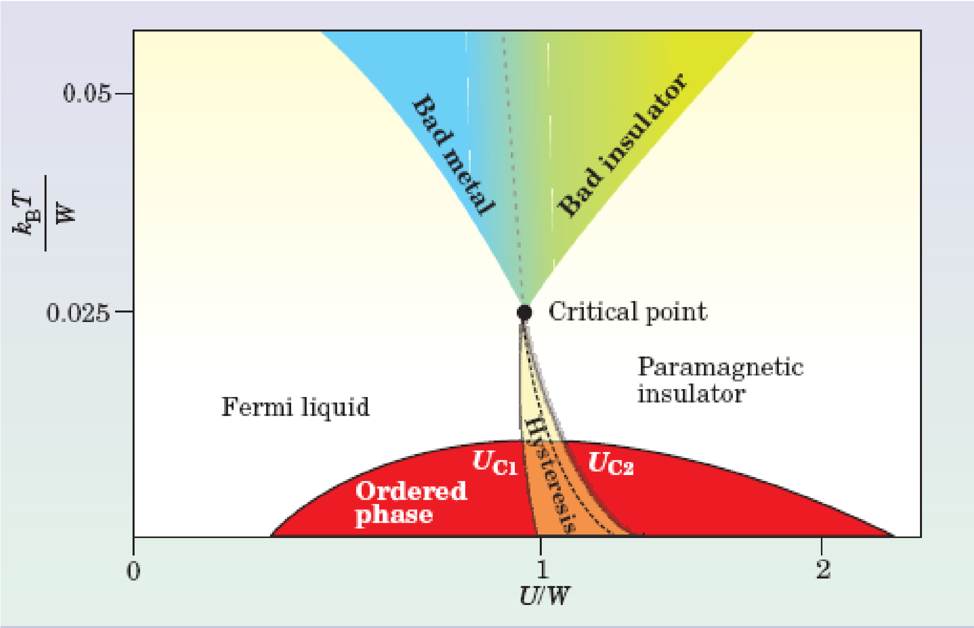
Localization-delocalization transition

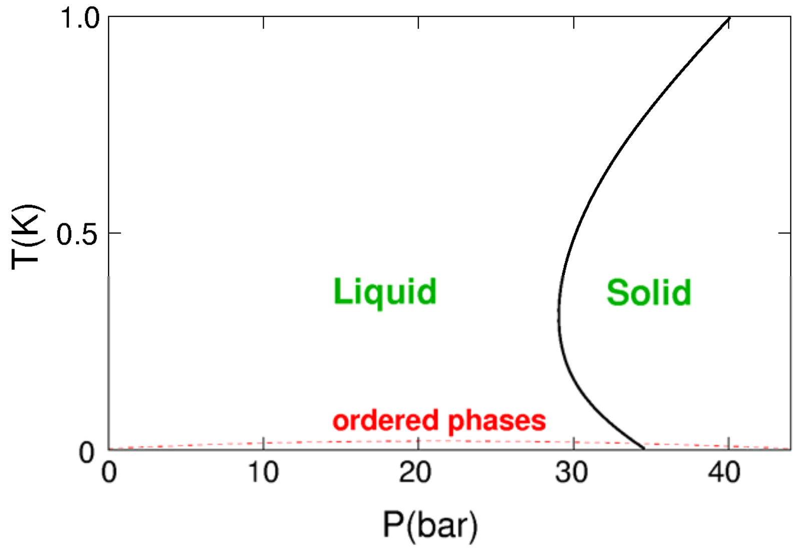
V_2O_3: Mott metal-insulator transition

^3He: liquid-solid transition









Strongly correlated electron materials:

V_2O_3, $NiSe_{2-x}S_x$, κ-organics, ...

Helium-3

Universality due to Fermi statistics + correlations

Peculiarities of Many-Particle Systems

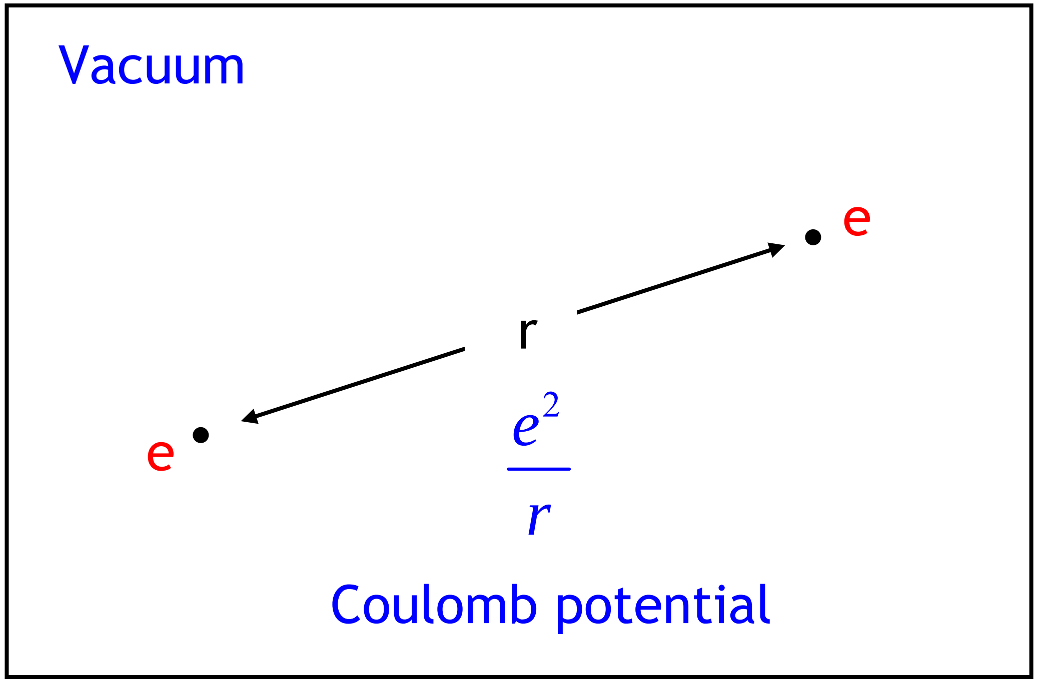
and "Emergence"

Interacting many-particle systems

Elementary ("bare") particles + interactions

$$\downarrow \; N \to \infty$$

effective ("quasi") particles + effective interactions



Interacting many-particle systems

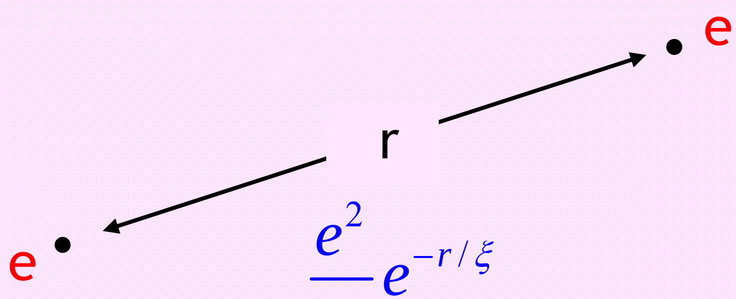
Elementary ("bare") particles + interactions

$$\Big\downarrow \; N \to \infty$$

effective ("quasi") particles + effective interactions

Electron gas: Screening

Simplest approximation: Thomas-Fermi



$$\frac{e^2}{r} e^{-r/\xi}$$

Effective Yukawa potential

Interacting many-particle systems

Elementary ("bare") particles + interactions

$$\downarrow \quad N \to \infty$$

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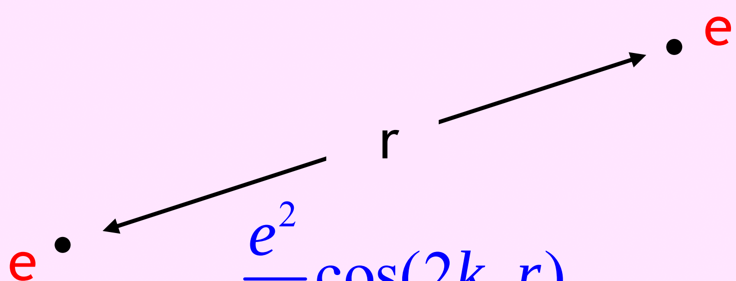
Electron gas: Screening

Better approximation: Lindhard



$$\frac{e^2}{r^3}\cos(2k_F r)$$

Friedel oscillations

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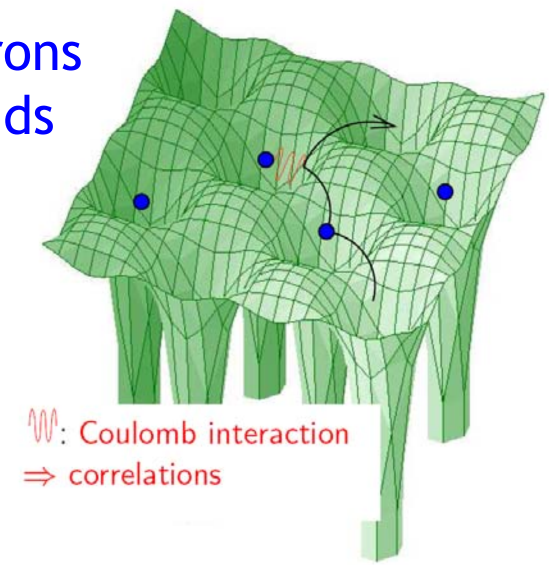
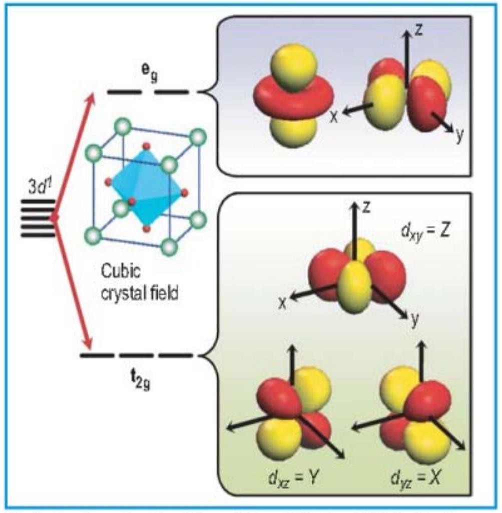
Electrons in solids





"Strong interaction" of electrons in localized orbitals

Interacting many-particle systems

Elementary ("bare") particles + interactions

$$\downarrow \; N \to \infty$$

effective ("quasi") particles + effective interactions

Entirely new phenomena arise, e.g., phase transitions

Strategy of Statistical Physics to detect Phase Transitions

Hamiltonian of many-body system	$H(V,N)$	
	$\downarrow$	
Energy of microstate *i*	$E_i(V,N)$	
	$\downarrow$	
Partition function	$Z(T,V,N)=\sum_i e^{-E_i/k_BT}$	
	$\downarrow$	
Free energy	$F(T,V,N)=-k_BT\ln Z$	
	$\downarrow$	
Chemical potential	$G(T,P,N)=N\mu(T,P)$	

Sum of analytic terms
→ Z analytic ?!

Thermodynamic limit:
$N,V\to\infty,\ N/V=const.$

→ infinitely many terms
→Z,μ can become
non-analytic

$\downarrow$ $\downarrow$ Lee, Yang (1952)

Singularity in $\mu^{(n)}(T,P)$ $\leftrightarrow$ **Phase transition**

Strategy of Statistical Physics to detect Phase Transitions

Hamiltonian of many-body system

$$H(V,N)$$

$$\downarrow$$

Energy of microstate *i*

$$E_i(V,N)$$

$$\downarrow$$

Partition function

$$Z(T,V,N) = \sum_i e^{-E_i/k_B T}$$

$$\downarrow$$

Free energy

$$F(T,V,N) = -k_B T \ln Z$$

$$\downarrow$$

Chemical potential

$$G(T,P,N) = N\mu(T,P)$$

Sum of analytic terms
→ Z analytic ?!

Thermodynamic limit:
$N,V \to \infty,\ N/V = const.$

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↓ Lee, Yang (1952)

Thermodynamic limit → unpredicted new phenomena

Interacting many-particle systems

$$\downarrow \; N \to \infty$$

Entirely new phenomena, e.g., phase transitions

$$\downarrow$$

Unpredicted "emergent" behavior

> We used to think that if we knew one, we knew two,
> because one and one are two.
> We are finding out that we must learn a great deal more about 'and'.
>
> Arthur Eddington (1882-1944)

"More is different" Anderson (1972)

Interacting many-particle systems

$$\downarrow \; N \rightarrow \infty$$

Emergence

Examples:

Superconductivity
Magnetism
Galaxy formation

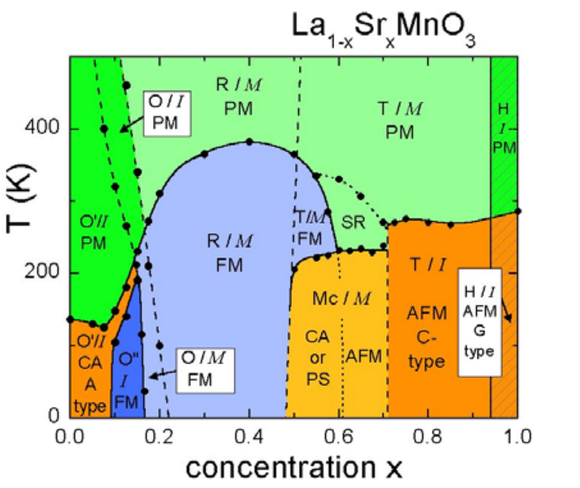
Traffic
Weather
Stock market

Ants
Human body
Consciousness

New Developments & Perspectives

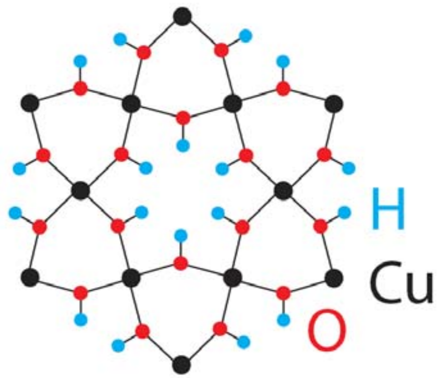
(i) Complex correlated electron materials

Explanation & prediction of properties of complex materials

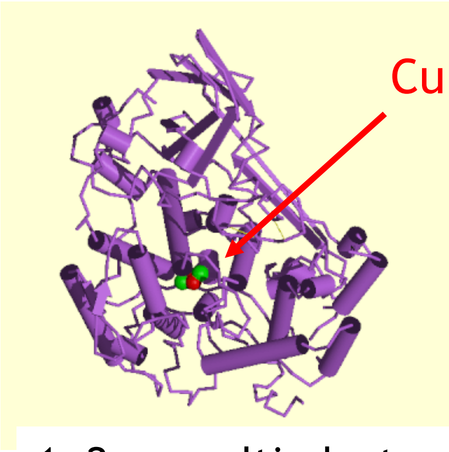


Phase diagram of $La_{1-x}Sr_xMnO_3$

Hemberger *et al.* (2002)



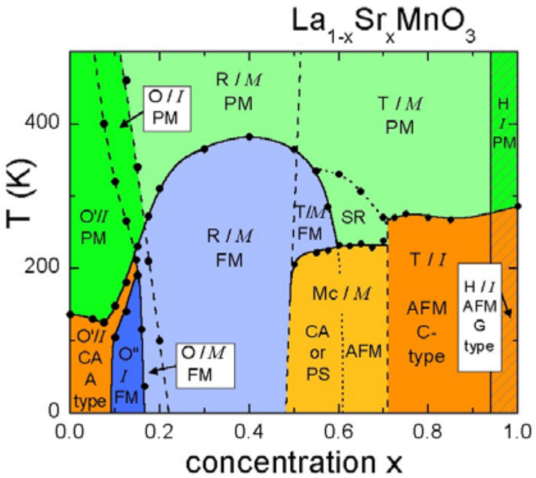
Kagome layer in $ZnCu_3(OH)_6Cl_2$ (herbertsmithite): Spin liquid behavior



1, 2, … multi-electron transfer in metalloprotein complexes
→ Photosynthesis

(i) Complex correlated electron materials

Explanation & prediction of properties of complex materials



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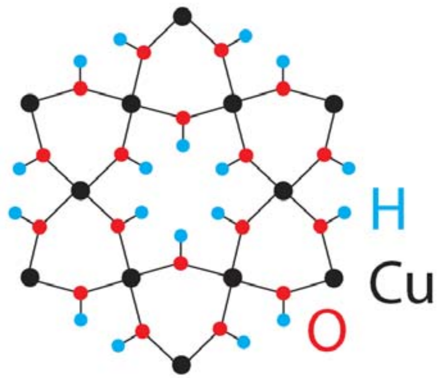

Kagome layer in
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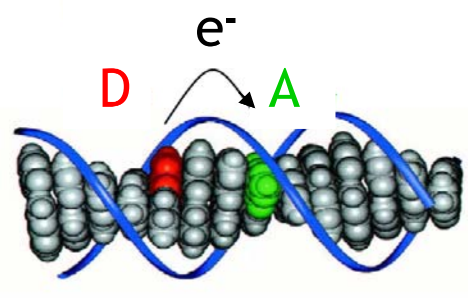
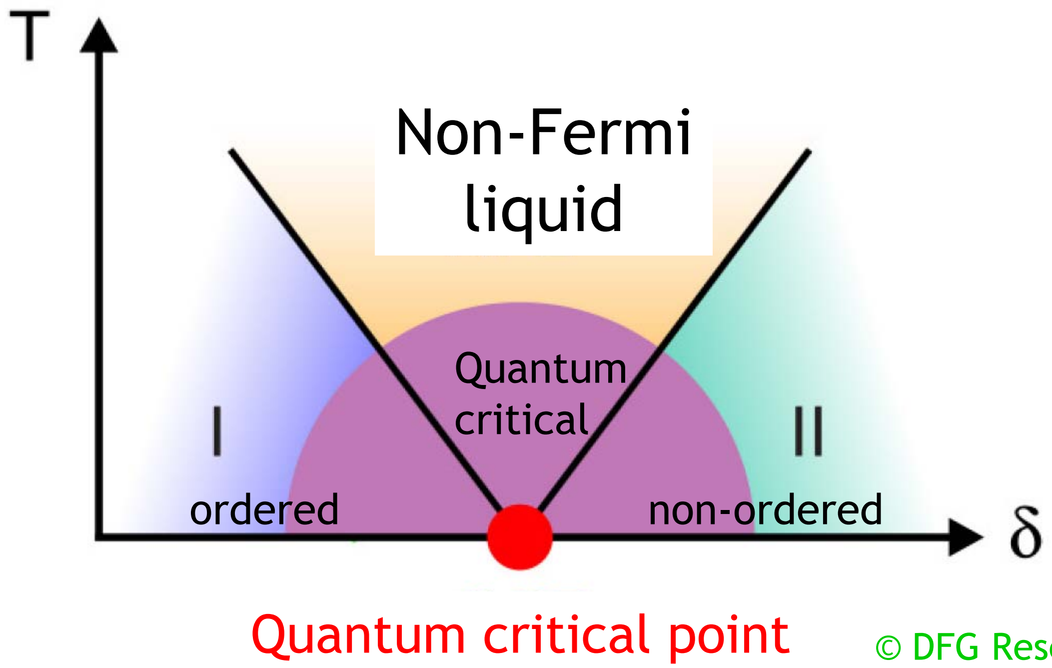


1, 2, … multi-electron transfer in DNA

→ damage & repair

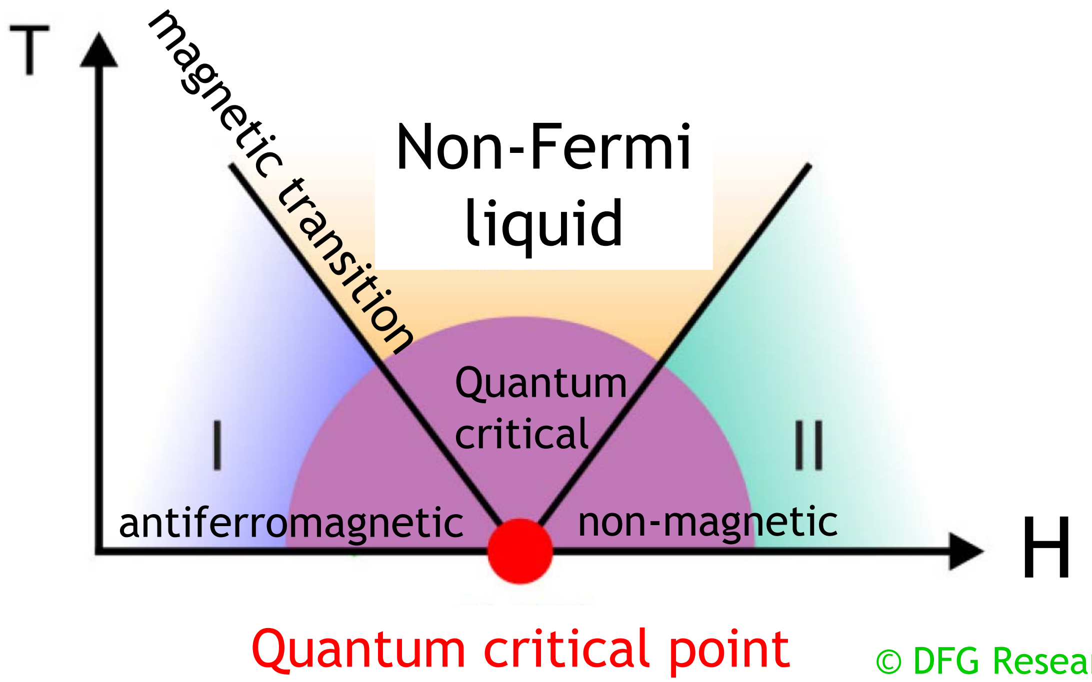
(ii) Quantum phase transitions



Driven by quantum fluctuations

- Non-Fermi liquid behavior
- Emergence of novel degrees of freedom
- New phases of matter

(ii) Quantum phase transitions



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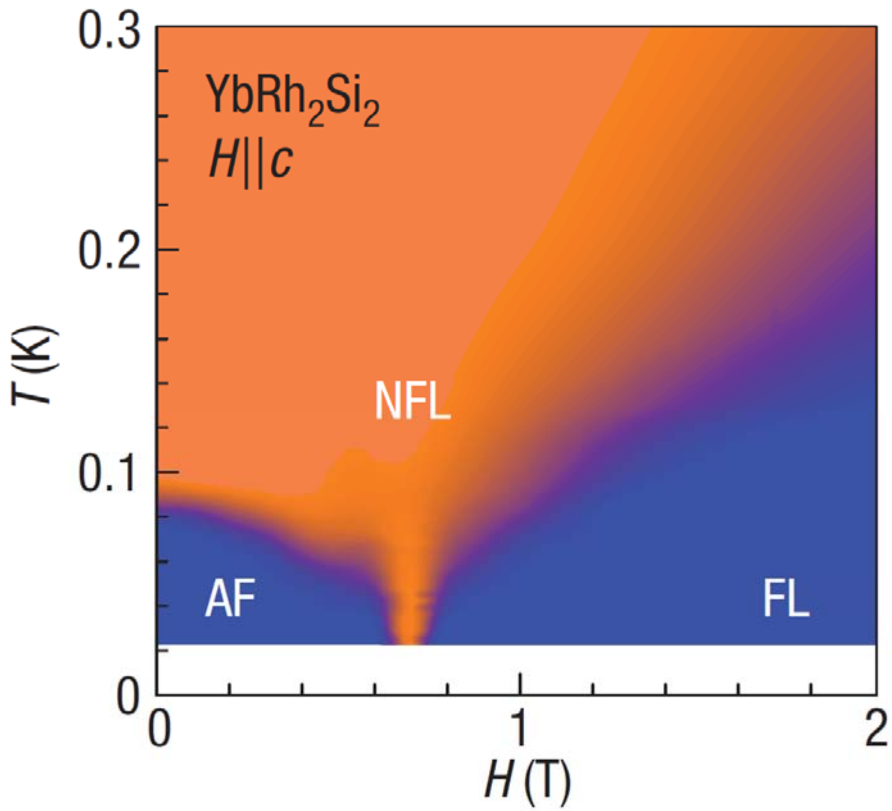


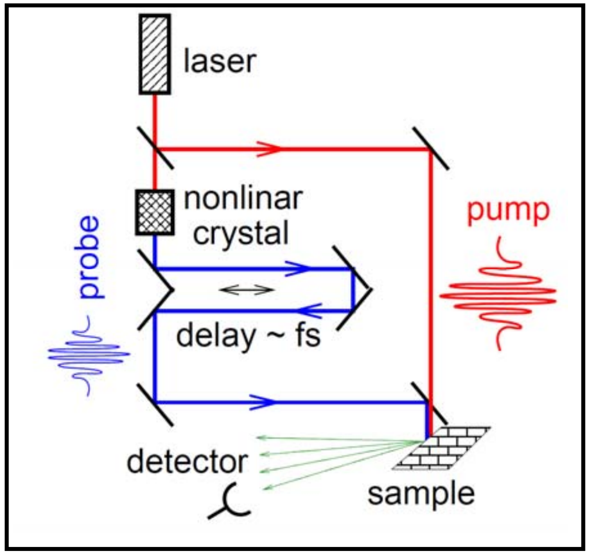
Custers *et al.* (2003)

Driven by quantum fluctuations

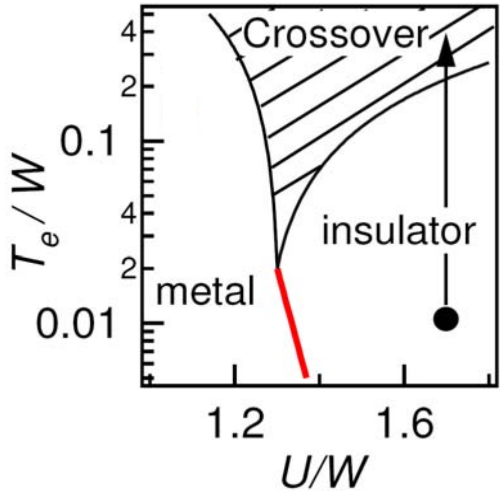
- Non-Fermi liquid behavior
- Emergence of novel degrees of freedom
- New phases of matter

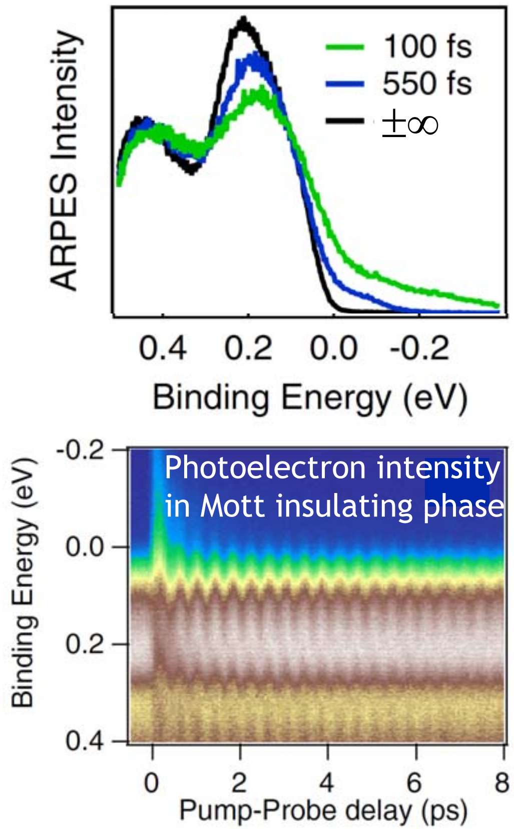
(iii) Correlated electrons in non-equilibrium

Real time evolution of correlation phenomena, e.g., time-resolved optical photoemission



Pump-probe experiment

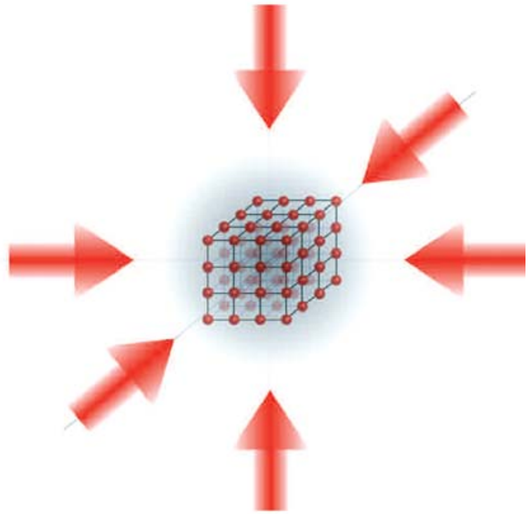




Perfetti *et al.* (2006)

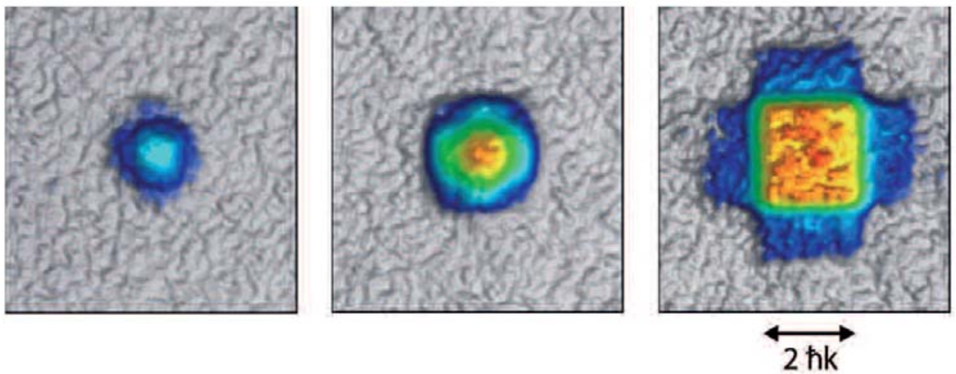
Required: Theory of non-equilibrium beyond linear response in correlated bulk materials

(iv) Correlated fermionic/bosonic atoms in optical lattices



Modugno et al. (2003)

Bosonic/fermionic atoms in optical lattices

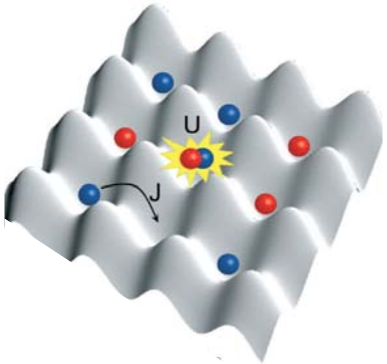


Köhl, Esslinger (2006)

Observation of Fermi surface (^{40}K atoms)

High degree of tunability: “Many-body tool box”

(iv) Correlated fermionic/bosonic atoms in optical lattices

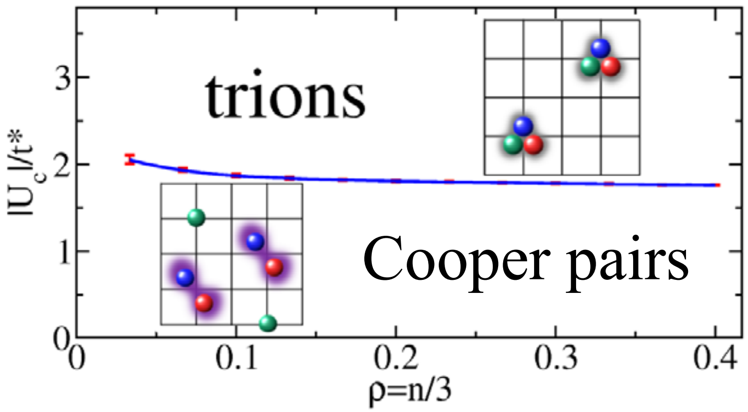


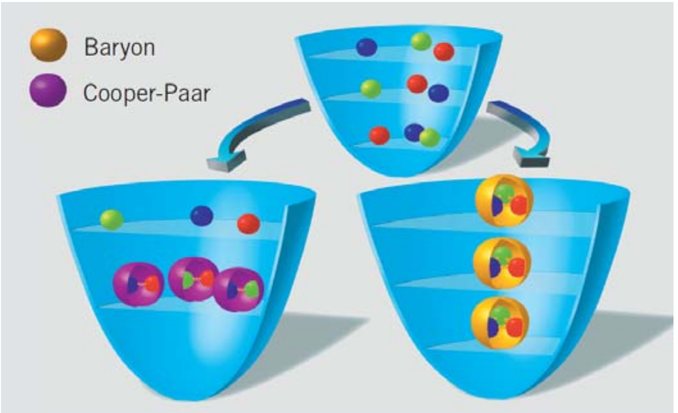
Hubbard model with ultracold atoms — Jaksch *et al.* (1998)

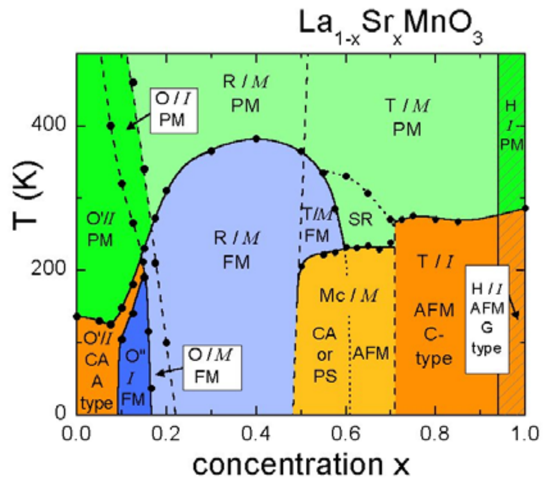
Angular momentum $L^{tot} = F$ → N=2F+1 hyperfine states

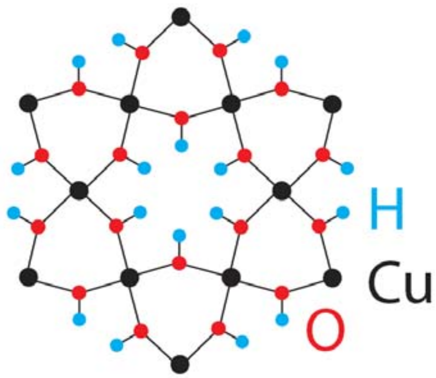
→ SU(N) Hubbard models — Honerkamp, Hofstetter (2004)

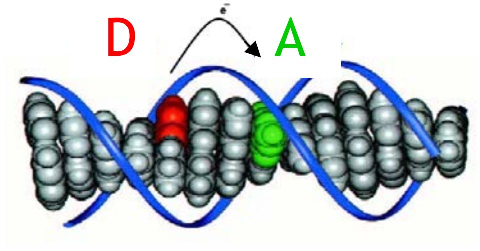
N=3, e.g. ^{6}Li, U<0: Color superconductivity, baryon formation (QCD)
Rapp *et al.* (2006)

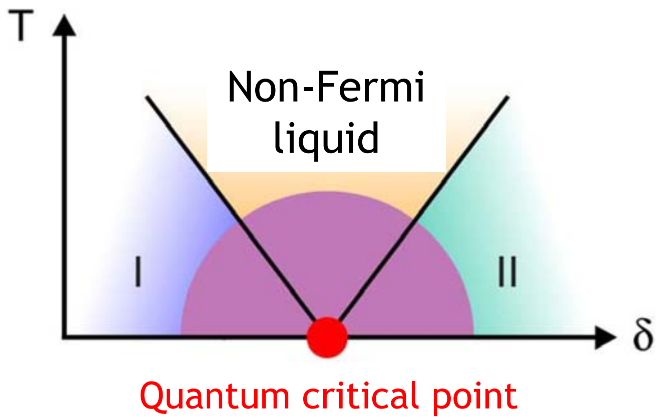


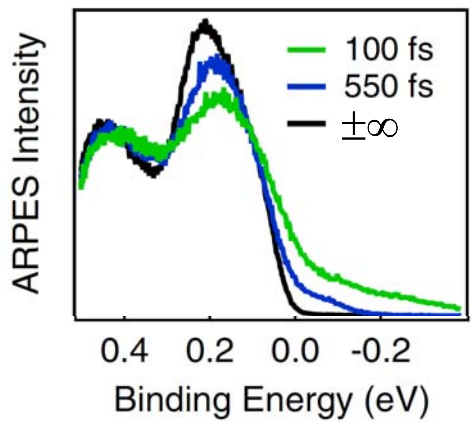


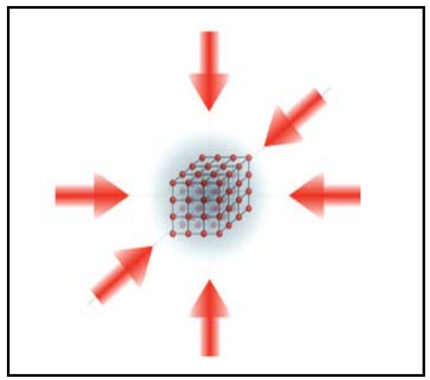












Correlated many-particle systems:
More manifold and fascinating than ever